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Effects of relative prices on contributions to the level and growth of real GDP

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Abstract

Existing procedures for GDP in chained or in constant prices ignore *relative prices* – ratios of industry GDP deflators to the economy’s GDP deflator – and, consequently, yield economically misleading results by *understating* (overstating) *level* contributions of industries with *above* (below) average relative prices, at the same time *understating* (overstating) *growth* contributions of industries with *rising* (falling) relative prices. These are illustrated by US GDP in chained prices and Philippine GDP in constant prices. However, the above misleading results could be mitigated by this paper’s *general* formulas for level and growth contributions applied to the same GDP. While allowing for differences and changes in relative prices, these general formulas encompass existing formulas as *special* cases of constant relative prices. In principle, relative prices convert real GDP of industries to the same (i.e., homogeneous) units so that they can be added to equal (i.e., additive) aggregate real GDP. Without relative prices – and, therefore, no homogeneity and no additivity – industry contributions to the level and growth of aggregate real GDP are questionable.

Key Words: Real GDP; relative prices; additivity; index numbers

JEL classification: C43; O47

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Introduction

This paper argues that real GDP of industries, as presently computed, are limited in use to studying industries *individually* or in *isolation* because they differ in units of measure due to different deflators. For this reason, they need to be converted to the same units – using relative prices as weights – for valid comparative analysis in a *group* setting, as in this paper, in determining and comparing industry contributions to the level and growth of the economy’s real GDP. Unfortunately, relative prices are ignored in existing procedures for real GDP in chained or in constant prices.

By definition, relative price is a ratio of one price to another where the price in the denominator may be chosen arbitrarily. However, since this paper is concerned with industry contributions to the level and growth of real GDP, it is appropriate for relative price to be the ratio of an industry’s GDP deflator to the overall GDP deflator. In this case, relative price is the *real price* per unit of an industry’s real GDP where overall real GDP is the *numeraire* or the common unit of measure. Analytically, relative prices are weights for converting different real GDP of industries into the same or homogeneous units that exactly add up to the economy’s real GDP. Without homogeneity and adding-up, industry contributions to the level and growth of the economy are questionable.

Hence, Section 2 of this paper presents general formulas for the level and growth of GDP in chained or in constant prices. It shows the effects of *differences* in relative prices between industries on their *level* contributions and the effects on their *growth* contributions of *changes* in relative prices that are separate from the effects of quantity changes. It is shown that existing formulas for these contributions are *special* cases of the above general formulas when relative prices are constant.

Section 3 applies this paper’s general formulas to US GDP in chained prices to show that existing “non-additivity” residuals are procedural in nature and not inherent in GDP in chained prices, contrary to current practice and prevailing theory (Balk, 2010; Ehemann, Katz, & Moulton, 2002; Whelan, 2002). It is shown that residuals in industry contributions to the level and growth of US GDP are due to differences and changes in relative prices that are ignored in present GDP procedures by the US Bureau of Economic Analysis. The general formulas are also

applied to Philippine GDP in constant prices to show that, although there are no residuals, contributions to the level and growth of GDP are, nevertheless, objectionable.

Furthermore, the empirical applications in Section 3 illustrate the analytic results in Section 2 that existing procedures yield misleading results by *understating* (overstating) the *level* contributions of industries with *above* (below) average relative prices while *understating* (overstating) the *growth* contributions of industries with *rising* (falling) relative prices. However, these misleading results could be mitigated by this paper's general formulas for the above industry contributions.

Section 4 shows that relative prices convert real GDP of industries in chained or in constant prices into "purchasing power parity" (PPP) values. In turn, this PPP conversion implies a *direct* formula for industry contributions to real GDP growth combining the growth effects of changes in relative prices and in quantities that were determined separately in Section 3.

Section 5 concludes this paper.

A general framework (GEN) for GDP

In period t , let there be nominal prices, p_{it}^j , and quantities, q_{it}^j , of $i = 1, 2, \dots, N$ final commodities (i.e., goods and services) where $j = 1, 2, \dots, M$ are mutually exclusive groups of "similar" commodities for aggregation purposes. Hence, $M < N$ since each j contains at least one i and some j contains more than one. GDP in current prices or nominal GDP is denoted by Y_t^j for group j (i.e., an industry) and by Y_t for the entire economy. By definition, noting that nominal GDP is additive,

$$Y_t^j \equiv \sum_i p_{it}^j q_{it}^j \quad ; \quad Y_t \equiv \sum_j Y_t^j = \sum_j \sum_i p_{it}^j q_{it}^j. \quad (1)$$

Published national accounts universally provide values of nominal GDP, Y_t and Y_t^j , as well as the corresponding real GDP, X_t and X_t^j . By definition, X_t is obtained by *dividing* Y_t by an aggregate GDP price index or deflator, $P_{0,t}$, that values X_t in prices of the base period 0. Industry real GDP, X_t^j , is similarly obtained from industry nominal GDP, Y_t^j , using the industry's GDP deflator, $P_{0,t}^j$. From the above definition of real GDP, the aggregate and industry GDP deflators may be obtained *implicitly* by

$$P_{0,t} \equiv \frac{Y_t}{X_t} \quad ; \quad P_{0,t}^j \equiv \frac{Y_t^j}{X_t^j}. \quad (2)$$

Combining (1) and (2) yields

$$P_{0,t} X_t = \sum_j P_{0,t}^j X_t^j \quad ; \quad X_t = \sum_j r_t^j X_t^j \quad ; \quad r_t^j \equiv \frac{P_{0,t}^j}{P_{0,t}}. \quad (3)$$

Since the results in (3) follow from the definitions in (1) and (2), they are valid regardless of the price index formulas underlying the GDP deflators (Dumagan, 2013). Therefore, (3) applies to GDP in *chained* prices if these deflators are *chained* Paasche price or Fisher price indexes or to GDP in *constant* prices if the deflators, $P_{0,t}$ and $P_{0,t}^j$, are *direct* Paasche price indexes (Balk, 2010). Since these price indexes exhaust the deflator formulas employed in existing GDP procedures, (3) is *perfectly general* and, thus, applies to real GDP of *all* countries.

It is important to note in (2) that the values of real GDP of industries, $X_t^j = Y_t^j / P_{0,t}^j$, are relevant for studying industries *individually* or in *isolation*. If this appears limiting, it is because the deflators, $P_{0,t}^j$, differ between industries so that the values of X_t^j do not have a common numeraire and, thus, differ in units of measure. However, this situation is corrected by applying relative price defined by $r_t^j \equiv P_{0,t}^j / P_{0,t}$, the ratio of each industry's GDP deflator to the aggregate GDP deflator, so that the economy's "real GDP basket" is the *numeraire*. Thus, r_t^j is the *real price* of each industry's real GDP, X_t^j , that converts them to the same unit of measure as aggregate real GDP, X_t .

It appears that relative prices in (3) are necessary to convert industry real GDPs to the same (i.e., homogeneous) units and to make them add up (i.e., additive) to aggregate real GDP as shown by $X_t = \sum_j r_t^j X_t^j = \sum_j Y_t^j / P_{0,t}$. This implies that without relative prices – and, therefore, no homogeneity and no additivity – analysis of industry contributions to the level and growth of aggregate real GDP would be questionable.

With due recognition, (3) probably first appeared in Tang and Wang's (2004) real GDP aggregation as basis for contributions to the level and growth of aggregate labor productivity (ALP). Following Tang and Wang (2004), relative prices have gained wider recognition (Diewert, 2015; Dumagan, 2014a; Dumagan, 2014b; Dumagan, 2013; Dumagan & Balk, 2016; Tang & Wang, 2014) in analyses of contributions to the level and growth of ALP or simply of

GDP in chained or in constant prices. The reason for the widening recognition is that – while Tang and Wang (2004) applied (3) to GDP in chained prices in Canada and the US, which are based on chained Fisher indexes – (3) is true regardless of the price index formula underlying the aggregate and industry GDP deflators.

For these reasons, (3) will be referred to as a *generalized* (GEN) real GDP level aggregation equation for the expository purposes of this paper.¹

Effects of relative prices on contributions to the level of real GDP

In concept, $P_{0,t}$ is an average of $P_{0,t}^j$ for all industries so that $P_{0,t}$ lies between the extreme values of $P_{0,t}^j$. Price indexes are strictly positive so that $\infty > r_t^j \equiv P_{0,t}^j/P_{0,t} > 0$ in practice but r_t^j should not be too far off above or below 1. This implies that relative prices cannot be dropped from $X_t = \sum_j r_t^j X_t^j$ unless *all* prices change in the same proportion (i.e., constant relative prices) in which case *all* price indexes are equal so that $r_{t-1}^j = r_t^j = 1$. Unless this condition holds, which is unlikely in practice, it follows that, in general,

$$X_t = \sum_j r_t^j X_t^j \quad ; \quad r_t^j \equiv \frac{P_{0,t}^j}{P_{0,t}} \neq 1 \quad ; \quad X_t \neq \sum_j X_t^j. \quad (4)$$

The “non-additivity” result in (4) is a well-known property of GDP in chained prices.² The implication of (4) is that non-additivity is the result of dropping r_t^j as if it equals 1, which is not necessarily true. Therefore, contrary to current practice and prevailing theory (Balk, 2010; Ehemann, Katz, & Moulton, 2002; Whelan, 2002), non-additivity of GDP in chained prices is procedural in nature and, hence, avoidable simply by implementing the first equation in (4).

An exception to (4) is the present procedure of aggregating GDP in constant prices *without* relative prices (i.e., $X_t = \sum_j X_t^j$). However, while true, it is arguable that this involves

¹ Dumagan (2013) coined the acronym GEAD for “generalized exactly additive decomposition” to refer to the Tang and Wang (2004) decomposition of aggregate labor productivity (ALP) growth by showing that their decomposition, which they applied to Canada and the US where GDP is in chained prices based on the chained Fisher price and quantity indexes, also applies to all other countries where GDP is in chained prices based on the chained Paasche price and Laspeyres quantity indexes or where GDP is in constant prices based on direct Paasche price and Laspeyres quantity indexes. GEAD yields the GEN framework in this paper when labor is eliminated from ALP.

² Non-additivity is universal in countries that have adopted GDP in chained prices. For some country practices, see Aspden (2000) for Australia; Chevalier (2003) for Canada; Maruyama (2005) for Japan; Brueton (1999) for the UK; Landefeld and Parker (1997) for the US; European Union (2007); and Schreyer (2004) for EU countries.

addition of *different* commodity baskets akin to the proverbial case of “adding apples and oranges” and, therefore, objectionable.

The result that $X_t = \sum_j X_t^j$ follows when the formulas for the GDP deflators $P_{0,t}^j$ and $P_{0,t}$ are *direct* Paasche price indexes. In this case, using the notation for prices and quantities in (1) and denoting the prices in the *fixed* base period 0 as p_{i0}^j , the direct Paasche price indexes are defined by

$$P_{0,t}^j \equiv \frac{\sum_i p_{it}^j q_{it}^j}{\sum_i p_{i0}^j q_{it}^j} \quad ; \quad P_{0,t} \equiv \frac{\sum_j \sum_i p_{it}^j q_{it}^j}{\sum_j \sum_i p_{i0}^j q_{it}^j}. \quad (5)$$

It follows from (1), (2), and (5) that real GDP of an industry and of the economy are given by³

$$X_t^j \equiv \frac{Y_t^j}{P_{0,t}^j} = \sum_i p_{i0}^j q_{it}^j \quad ; \quad X_t \equiv \frac{Y_t}{P_{0,t}} = \sum_j \sum_i p_{i0}^j q_{it}^j = \sum_j X_t^j. \quad (6)$$

Real GDP above is in *constant* prices from the fact that current quantities, q_{it}^j , are valued at the same prices of the fixed base period, p_{i0}^j .

However, although $X_t = \sum_j X_t^j$ is true for GDP in constant prices as shown in (6), this result cannot be the rule because $X_t = \sum_j r_t^j X_t^j$ is also true in this case considering that (2), (3), and (5) yield

$$X_t \equiv \frac{Y_t}{P_{0,t}} = \sum_j \sum_i p_{i0}^j q_{it}^j \quad ; \quad \sum_j r_t^j X_t^j = \frac{1}{P_{0,t}} \sum_j P_{0,t}^j X_t^j = \sum_j \sum_i p_{i0}^j q_{it}^j. \quad (7)$$

It follows from (6) and (7) that

$$X_t = \sum_j r_t^j X_t^j = \sum_j X_t^j = \sum_j \sum_i p_{i0}^j q_{it}^j. \quad (8)$$

Therefore, the economy’s aggregate real GDP in constant prices is the same with or without relative prices as weights of industry real GDP. However, the use of relative prices as weights is not a matter of indifference because it is analytically necessary as argued below.

Nominal GDP is additive (i.e., $Y_t = \sum_j Y_t^j$) from the fact that a unit of Y_t^j is the *same* for all j since Y_t^j is “money.” Additivity of nominal GDP must translate to additivity of real GDP for logical consistency. This requirement is satisfied by $X_t = \sum_j r_t^j X_t^j$ where, as noted earlier,

³ The result that $X_t = \sum_j X_t^j$ follows from the “consistency-in-aggregation” property of the direct Paasche price and direct Laspeyres quantity indexes (Balk, 1996; Diewert, 1978; Vartia, 1976).

$r_t^j \equiv P_{0,t}^j/P_{0,t}$ is the real price of each industry's real GDP in units of the economy's "real GDP basket" as numeraire. In effect, relative prices are conversion factors that make real GDP homogeneous across industries so that the "addition" in $X_t = \sum_j r_t^j X_t^j = r_t^1 X_t^1 + r_t^2 X_t^2 + \dots + r_t^M X_t^M = \sum_j Y_t^j/P_{0,t} = Y_t/P_{0,t}$ is legitimate because the elements being added are in the same units as X_t .

In contrast, although the "equality" is true in the case of GDP in constant prices that $X_t = \sum_j X_t^j = X_t^1 + X_t^2 + \dots + X_t^M$, the "addition" is problematic. Two industries suffice to illustrate the problem with the addition (i.e., $X_t^1 + X_t^2 = Y_t^1/P_{0,t}^1 + Y_t^2/P_{0,t}^2$). If "1" represents the apple industry where Y_t^1 is the nominal value of apples and $P_{0,t}^1$ is the direct Paasche price index of apples then $X_t^1 = Y_t^1/P_{0,t}^1$ is measured in "baskets of apples." In similar fashion, if "2" represents the orange industry, then $X_t^2 = Y_t^2/P_{0,t}^2$ is measured in "baskets of oranges." Thus, in general, $X_t = \sum_j X_t^j = X_t^1 + X_t^2 + \dots + X_t^M$ involves addition of deflated values representing quantity bundles with no common numeraire. Moreover, $X_t = \sum_j X_t^j$ is equivalent to $Y_t/P_{0,t} = \sum_j Y_t^j/P_{0,t}^j$ that appears to violate the additivity of nominal GDP because it does not necessarily imply $Y_t = \sum_j Y_t^j$ considering that $P_{0,t} = P_{0,t}^j$, all j , is not necessarily true except in the base period 0 when all price indexes equal 1.

The preceding analysis implies that relative prices should not be ignored in the GEN *level* equation for real GDP in (3). Noting that an industry's contribution to the level of X_t is $r_t^j X_t^j$, ignoring r_t^j *understates* the level contributions of industries with *above* average prices or $r_t^j > 1$ and, conversely, *overstates* the level contributions of industries with *below* average prices or $0 < r_t^j < 1$. These results apply to present practices of GDP in chained or in constant prices.

Effects of relative prices on contributions to growth of real GDP

Consider that (3) applies generally so that $X_{t-1} = \sum_j r_{t-1}^j X_{t-1}^j$ is true. It follows that the relative change in real GDP is

$$\frac{X_t}{X_{t-1}} = \sum_j r_t^j w_{t-1}^j \frac{X_t^j}{X_{t-1}^j} \quad ; \quad w_{t-1}^j \equiv \frac{X_{t-1}^j}{X_{t-1}}. \quad (9)$$

It may be recognized that X_t/X_{t-1} is the *implicit* aggregate GDP quantity index while X_t^j/X_{t-1}^j is an *implicit* industry GDP quantity index. Thus, (9) states that the implicit aggregate GDP

quantity index equals the weighted sum of implicit industry GDP quantity indexes where the industry weights are given by $r_t^j w_{t-1}^j$ that do not necessarily sum to 1 unless relative prices are constant (i.e., $r_{t-1}^j = r_t^j$) as shown below.

Note that $\sum_j w_{t-1}^j$ is not necessarily equal to 1 from (4). However, (1), (2), (3), and (9) yield

$$r_{t-1}^j w_{t-1}^j = \left(\frac{Y_{t-1}^j / X_{t-1}^j}{Y_{t-1} / X_{t-1}} \right) \left(\frac{X_{t-1}^j}{X_{t-1}} \right) = \frac{Y_{t-1}^j}{Y_{t-1}} \quad ; \quad \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} = 1. \quad (10)$$

Using (10), it can be verified that (9) yields

$$\frac{X_t}{X_{t-1}} - 1 = \sum_j \left[\frac{Y_{t-1}^j}{Y_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right) + \frac{X_t^j}{X_{t-1}} (r_t^j - r_{t-1}^j) \right]. \quad (11)$$

The result in (11) is a *generalized* (GEN) formula for the *growth* rate of the economy's GDP in chained or in constant prices. This formula shows that the economy's real GDP growth equals the sum of industry growth contributions where each contribution has two parts. One part is

$$\text{PGE (pure growth effect)} \equiv \frac{Y_{t-1}^j}{Y_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right). \quad (12)$$

PGE is an industry's GDP growth – from a change in quantities – weighted by its nominal GDP share. The other is

$$\text{PCE (price change effect)} \equiv \frac{X_t^j}{X_{t-1}} (r_t^j - r_{t-1}^j). \quad (13)$$

PCE comes from a change in the industry's relative price r_{t-1}^j to r_t^j .⁴ Unfortunately, PCE is not computed in present practice because relative prices are ignored as already noted. This is discussed further below.

Since (9) comes from (3) that by definition is true for *any* real GDP, it follows that the aggregate real GDP growth equation in (11) is “*exact*” (i.e., no residual). This means that when applied to data on GDP in chained or in constant prices, (11) equals the “actual” GDP growth rate. This analytic result is confirmed empirically by the applications presented later in this paper.

Comparing GEN procedures to those in current practice

⁴ It may be noted that PGE in (12) is the same as PGE in Dumagan (2014a and 2014b) while PCE in (13) is the sum of GPIE (growth-price interaction effect) and RPE (relative price effect) also in Dumagan (2014a and 2014b). That is, except for differences in notation, it can be verified that PCE = GPIE + RPE.

The official BEA formula (Moulton & Seskin, 1999) for an industry's growth contribution is the formula for a component's contribution to growth of the Fisher quantity index that underpins US real GDP. BEA's formula is also "exact" in that the sum of growth contributions equals growth of the Fisher quantity index. However, by construction, quantity indexes have the *same* prices in the numerator and denominator of the index.⁵ Thus, the exactness of BEA's formula is different from the exactness of this paper's GEN formula in (11), which holds regardless of the quantity index formula underlying GDP while allowing for changes in relative prices.

As noted by Balk (2004), BEA's formula may be traced back to Van IJzeren (1952) and is mathematically equivalent to a more recent derivation by Dumagan (2002) of the "additive" decomposition of the growth of the Fisher quantity index. Using the latter for comparison, it can be shown that BEA's formula is *approximately* equal to PGE in (12). One source of difference is that, while the weights in BEA's formula also sum to 1, each weight is approximately equal to the industry's share in nominal GDP, Y_{t-1}^j/Y_{t-1} . The other source of difference is due to the fact that the *explicit* (i.e., formula) Fisher index is not consistent in aggregation (Diewert, 1978). This means that, in contrast to (9), the *explicit* aggregate GDP Fisher quantity index cannot be expressed as the weighted sum of the *explicit* industry GDP Fisher quantity indexes. This property technically forces BEA to compute contributions to growth starting at the lowest level, (i.e., at the commodity level i) and then sum them to the industry level j . But, as shown empirically later, the above differences between BEA's industry growth contributions and PGE in (12) only amount to rounding off errors because they become equal when rounded off to two decimal places.

In the above light, BEA's formula does not capture growth effects of relative price changes and, thus, PCE in (13) constitutes BEA's growth residual. This implies that BEA *understates* growth contributions of industries with *rising* relative prices (i.e., $r_t^j - r_{t-1}^j > 0$) and, conversely, *overstates* growth contributions of industries with *falling* relative prices (i.e., $r_t^j - r_{t-1}^j < 0$).

⁵ The Fisher index (Fisher, 1922) is the geometric mean of Laspeyres and Paasche indexes. The Laspeyres quantity index uses the same prices in $t-1$ (in the numerator and denominator) while the Paasche quantity index uses the same prices in t . Hence, the Fisher quantity index holds prices the same at the "average" of the prices in the two periods.

Finally, suppose relative prices are constant (i.e., $r_{t-1}^j = r_t^j = 1$) for all j and t , then the GEN formula in (3) for level contributions becomes $X_t = \sum_j X_t^j$, which is the GDP level formula in current practice. Moreover, by using (10) when $r_{t-1}^j = 1$, the GEN formula in (11) for growth contributions becomes $(X_t/X_{t-1}) - 1 = \sum_j \{w_{t-1}^j [(X_t^j/X_{t-1}^j) - 1]\}$, which is the GDP growth formula in current practice. Thus, this paper's GEN framework encompasses existing procedures as special cases of constant relative prices.

Applications of the general (GEN) framework

This paper's GEN framework for industry contributions to the level and growth of GDP is applicable to US GDP in chained prices and to Philippine GDP in constant prices. These applications substantiate the preceding analytic results.

GEN application to US GDP in chained prices

Consider US GDP in Table 1. GDP in current dollars is additive so that the zero residuals imply that there are *no* missing industries.⁶ It can be verified from Table 1 that $r_{t-1}^j \neq 1$ and $r_t^j \neq 1$. Therefore, $X_{t-1} = \sum_j r_{t-1}^j X_{t-1}^j$ and $X_t = \sum_j r_t^j X_t^j$ imply that if relative prices are ignored, then for industries with $r_{t-1}^j, r_t^j > 1$ (i.e., above average prices) the level contributions are understated while for those with $0 < r_{t-1}^j, r_t^j < 1$ (i.e., below average prices) the level contributions are overstated. These results show that residuals from "non-additivity" (i.e., $X_{t-1} \neq \sum_j X_{t-1}^j$ and $X_t \neq \sum_j X_t^j$) are due to *ignoring* relative prices. That is, these residuals are procedural and not inherent in GDP in chained prices.

The results of the applications of PGE in (12) and of PCE in (13) to US GDP in Table 1 are presented in Table 2 under the heading GEN. It is interesting to note that for the same industry BEA's growth contribution equals PGE when PGE is rounded off to two decimal places. This confirms the earlier discussion that BEA's growth contribution captures almost solely the effects of quantity growth like PGE. Therefore, BEA's growth contribution almost totally excludes PCE. For all industries, BEA yields 2.15 percent while PGE + PCE = 2.43

⁶ Following present BEA procedures, the values of the residuals in chained dollars in Table 1 are sensitive to a number of factors such as the level of detail of the industries. However, given that the residuals in current dollars are zero, this paper's procedure in (3) implies that the residuals in chained dollars should be zero because $Y_t = \sum_j Y_t^j$ necessarily implies $X_t = \sum_j r_t^j X_t^j$ no matter the number of industries or the definition of j .

percent, the “actual” 2014 US GDP growth.⁷ In general, (PGE + PCE) exactly equals actual GDP growth.

Table 2 shows *positive* (negative) PCE for industries with *rising* (falling) relative prices. Therefore, by excluding PCE, BEA *understates* (overstates) the growth contributions of industries with *rising* (falling) relative prices. BEA’s exclusions of PCE could result in sign reversals of growth contributions, in the case of agriculture, forestry, fishing, and hunting and utilities. As shown, the growth contribution of utilities switches from positive (0.035), according to GEN, to negative (−0.07), according to BEA. Hence, excluding PCE could make BEA’s contributions misleading.

Table 1. US GDP level and growth					
	BEA				
	GDP in current Prices		GDP in chained Prices		GDP growth
	(billions of current dollars)		(billions of chained 2009 dollars)		(percent)
	2013	2014	2013	2014	2014
US GDP level and growth	16,663.0	17,348.2	15,583.3	15,961.7	2.43
Contributions to US GDP level and growth					
Agriculture, forestry, fishing, and hunting	225.4	215.4	145.8	149.6	0.04
Mining	441.0	453.8	335.3	358.7	0.18
Utilities	270.5	280.8	275.9	264.8	-0.07
Construction	619.9	664.0	584.1	589.6	0.04
Durable goods	1,082.0	1,125.5	1,078.3	1,095.9	0.11
Nondurable goods	942.6	972.2	789.3	801.9	0.09
Wholesale trade	1,002.2	1,044.5	919.3	950.1	0.20
Retail trade	967.6	997.8	907.7	924.0	0.10
Transportation and warehousing	483.5	505.7	442.0	445.7	0.02
Information	793.8	824.7	795.4	826.2	0.18
Finance and insurance	1,150.2	1,222.9	994.1	1,016.7	0.16
Real estate and rental and leasing	2,145.3	2,247.7	2,052.0	2,100.7	0.31
Professional, scientific, and technical services	1,136.6	1,193.0	1,071.6	1,107.3	0.23
Management of companies and enterprises	322.0	337.9	313.7	335.3	0.13
Administrative and waste management services	493.8	526.0	483.0	503.8	0.13
Educational services	184.7	192.8	164.2	167.4	0.02
Health care and social assistance	1,188.5	1,226.9	1,117.5	1,141.6	0.15
Arts, entertainment, and recreation	164.3	172.4	157.2	161.8	0.03
Accommodation and food services	461.4	488.0	431.7	444.8	0.08
Other services, except government	363.1	381.6	327.5	335.6	0.05
Federal government	708.4	718.0	661.9	656.1	-0.04
State and local government	1,516.2	1,556.6	1,388.7	1,390.9	0.01
Residuals: "Not allocated by industry"	0	0	123.9	164.9	0.28

Source: Bureau of Economic Analysis (BEA), released on November 5, 2015.

⁷ Although the data are different, the formula for PGE in Table 2 is the same as that for PGE in Table 3 of Dumagan (2014a) where the formula for the sum of GPIE (growth-price interaction effect) and RPE (relative price effect) equals that for PCE in Table 2 above.

Table 2. Industry contributions to US GDP growth				
	BEA	GEN		Actual
	GDP growth	PGE	PCE	GDP growth
	(percent)	(percent)	(percent)	(percent)
	2014	2014	2014	2014
		(1)	(2)	(1)+(2)
Contribution to GDP growth (percentage point)				
Agriculture, forestry, fishing, and hunting	0.04	0.035	-0.116	-0.081
Mining	0.18	0.185	-0.152	0.033
Utilities	-0.07	-0.065	0.100	0.035
Construction	0.04	0.035	0.165	0.200
Durable goods	0.11	0.106	0.046	0.152
Nondurable goods	0.09	0.090	-0.007	0.083
Wholesale trade	0.20	0.202	-0.049	0.152
Retail trade	0.10	0.104	-0.020	0.084
Transportation and warehousing	0.02	0.024	0.060	0.084
Information	0.18	0.184	-0.079	0.105
Finance and insurance	0.16	0.157	0.161	0.318
Real estate and rental and leasing	0.31	0.306	0.091	0.396
Professional, scientific, and technical services	0.23	0.227	-0.005	0.223
Management of companies and enterprises	0.13	0.133	-0.070	0.063
Administrative and waste management services	0.13	0.128	0.015	0.142
Educational services	0.02	0.022	0.008	0.030
Health care and social assistance	0.15	0.154	-0.042	0.111
Arts, entertainment, and recreation	0.03	0.029	0.003	0.032
Accommodation and food services	0.08	0.084	0.028	0.112
Other services, except government	0.05	0.054	0.020	0.074
Federal government	-0.04	-0.037	0.025	-0.012
State and local government	0.01	0.014	0.077	0.091
Sum	2.15	2.17	0.26	2.43
US GDP percent growth	2.43			2.43
Residuals: "Not allocated by industry"	0.28			0.00

Source: BEA results are copied from Table 1 while GEN results are the author's calculations of PGE in (12) and PCE in (13) using the data in Table 1.

GEN application to Philippine GDP in constant prices

It was shown in (8) that GDP in constant prices is additive with or without relative prices. Therefore, in general,

$$X_{t-1} = \sum_j r_{t-1}^j X_{t-1}^j = \sum_j X_{t-1}^j \quad ; \quad X_t = \sum_j r_t^j X_t^j = \sum_j X_t^j. \quad (14)$$

It follows from (11) and (14) that there are two ways of computing industry contributions to the growth of GDP in constant prices given by

$$\frac{X_t}{X_{t-1}} - 1 = \sum_j \left[\frac{Y_{t-1}^j}{Y_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right) + \frac{X_t^j}{X_{t-1}} (r_t^j - r_{t-1}^j) \right] = \sum_j \frac{X_{t-1}^j}{X_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right). \quad (15)$$

It may be noted that the right-hand side of (15) is the TRAD formula for growth contributions to GDP in constant prices. Using the definition of r_{t-1}^j in (3), this formula can be rewritten as

$$\frac{X_t}{X_{t-1}} - 1 = \sum_j \frac{X_{t-1}^j}{X_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right) = \sum_j \frac{Y_{t-1}^j / Y_{t-1}}{r_{t-1}^j} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right). \quad (16)$$

The GEN formula for an industry's contribution to GDP growth is reproduced in the middle of (15) where PGE (pure growth effect) is given by the first term and PCE (price change effect) is given by the second term. For comparison, the TRAD formula for an industry's contribution to the growth of GDP in constant prices (NEDA, 2011) is given in the right-hand side of (15) and (16) by

$$\text{TRAD (traditional)} \equiv \frac{X_{t-1}^j}{X_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right) = \frac{Y_{t-1}^j / Y_{t-1}}{r_{t-1}^j} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right) = \frac{\text{PGE}}{r_{t-1}^j}. \quad (17)$$

The results of applying TRAD, PGE, and PCE to Philippine GDP in Table 3 are presented in Table 4.⁸ It is important to note in (15) and in Table 4 that the sum of TRAD necessarily equals the sum of (PGE + PCE) for all industries and also equals the “actual” 5.81% GDP growth in 2015. However, TRAD may differ from (PGE + PCE) for each industry as explained below.

It turns out in (17) that $\text{TRAD} = \text{PGE} / r_{t-1}^j$ where $r_{t-1}^j \equiv P_{0,t-1}^j / P_{0,t-1}$ and $P_{0,t-1}$ is the average of $P_{0,t-1}^j$ of all industries. Hence, for industries with above average relative prices or $r_{t-1}^j > 1$, $\text{TRAD} < \text{PGE}$. In contrast, for those with below average relative prices or $0 < r_{t-1}^j < 1$, $\text{TRAD} > \text{PGE}$. That is, TRAD understates (overstates) the growth contributions of industries with above (below) average relative prices.

PCE captures the growth effects of relative price changes from r_{t-1}^j to r_t^j that TRAD ignores. Hence, TRAD could yield a positive growth contribution when (PGE + PCE) is negative. This is shown in Table 4 by agriculture and forestry. This industry had a negative PCE that more than offset the positive PGE to end up with a negative (−0.432) overall growth contribution but TRAD showed a positive (0.054) growth contribution. Thus, TRAD could yield misleading results.

⁸ Except for differences in data, the formula for PGE in Table 4 is the same as that for PGE in Table 4 of Dumagan (2014b) where the formula for the sum of GPIE (growth-price interaction effect) and RPE (relative price effect) equals that for PCE in Table 4 above.

	GDP in current prices		GDP in constant prices		GDP growth
	(million current pesos)		(million constant 2000 pesos)		(percent)
	2014	2015	2014	2015	2015
Philippines	12,642,735	13,285,239	7,164,016	7,579,941	5.81
Agriculture and forestry	1,230,996	1,168,282	587,329	591,215	0.66
Fishing	197,134	195,653	130,495	128,109	-1.83
Mining and quarrying	125,390	103,826	76,474	75,444	-1.35
Manufacturing	2,603,644	2,669,622	1,666,514	1,762,103	5.74
Construction	828,161	913,761	422,150	459,586	8.87
Electricity gas and water supply	411,701	416,579	229,555	240,625	4.82
Transport communication and storage	783,492	854,259	536,562	579,054	7.92
Trade and repair of vehicles, personal, and household goods	2,243,271	2,401,777	1,184,994	1,266,656	6.89
Financial intermediation	988,894	1,060,471	515,484	545,076	5.74
Real estate renting and business activity	1,553,387	1,714,102	803,241	861,581	7.26
Public administration, defense, and social security	503,110	506,600	292,441	294,229	0.61
Other services	1,173,555	1,280,307	718,777	776,263	8.00

Source: Economic and Social Database (04-06-2016), Philippine Institute for Development Studies from the National Accounts, Gross Domestic Product by Industrial Origin (Revised/Rebased), National Statistical Coordination Board.

“Purchasing power parity” in the GEN Framework

Recall from (3) that GEN aggregate GDP is $X_t = \sum_j r_t^j X_t^j$ where the industry level contribution is $r_t^j X_t^j = (P_{0,t}^j/P_{0,t})(Y_t^j/P_{0,t}^j) = Y_t^j/P_{0,t}$. Thus, the GDP deflator, $P_{0,t}$, converts GDP of industries to “exchange value parity.” This GEN feature may appear new but conceptually is not because it is *similar* to converting GDP of countries to “purchasing power parity” (PPP) as explained below.

	TRAD	GEN		Actual
	GDP growth	PGE	PCE	GDP growth
	(percent)	(percent)	(percent)	(percent)
	2015	2015	2015	2015
		(1)	(2)	(1)+(2)
Industry contributions to GDP growth (percentage point)				
Agriculture and forestry	0.054	0.064	-0.497	-0.432
Fishing	-0.033	-0.029	0.027	-0.001
Mining and quarrying	-0.014	-0.013	-0.152	-0.165
Manufacturing	1.334	1.181	-0.514	0.667
Construction	0.523	0.581	0.146	0.727
Electricity gas and water supply	0.155	0.157	-0.096	0.061
Transport communication and storage	0.593	0.491	0.116	0.606
Trade & repair of vehicles, personal, & household goods	1.140	1.223	0.162	1.385
Financial intermediation	0.413	0.449	0.175	0.624
Real estate renting and business activity	0.814	0.892	0.472	1.365
Public administration, defense, and social security	0.025	0.024	0.031	0.055
Other services	0.802	0.742	0.172	0.914
Sum = Philippine GDP percent growth	5.81	5.76	0.04	5.81

Source: Author's calculations of PGE in (12), PCE in (13), and TRAD in (20) using the data in Table 3.

Suppose US nominal GDP is $\$Y^S$ and GDP deflator is P^S so that US real GDP is $\$Y^S/P^S$. Also, suppose UK nominal GDP is $\pounds Y^K$ and GDP deflator is P^K so that UK real GDP is $\pounds Y^K/P^K$. The sum $\$Y^S/P^S + \pounds Y^K/P^K$ is not sensible because the units are different. To make the sum sensible, it may be expressed in US PPP by multiplying $\pounds Y^K/P^K$ by the “real exchange rate” (RER) to yield

$$\frac{\$Y^S}{P^S} + \frac{\pounds Y^K}{P^K} \left(\frac{P^K}{P^S} \right) \left(\frac{\$}{\pounds} \right) = \frac{\$Y^S}{P^S} + \frac{\pounds Y^K}{P^K} \left(\frac{\$/P^S}{\pounds/P^K} \right) = \frac{\$Y^S}{P^S} + \frac{\$Y^K}{P^S}. \quad (18)$$

In (18), $(\$/P^S)/(\pounds/P^K)$ is the RER that adjusts the nominal exchange rate, $\$/\pounds$, for differences in purchasing power (i.e., difference between P^S and P^K). Thus, RER converts UK real GDP to the same units as US real GDP. The result in (18) is that they have the same real exchange value, $(\$/P^S)/(\pounds/P^K) = 1$, which demonstrates PPP.⁹

Following the preceding example, the GEN industry level contribution given by $r_t^j X_t^j = (P_{0,t}^j/P_{0,t})(Y_t^j/P_{0,t}^j) = Y_t^j/P_{0,t}$ is conceptually similar to a PPP value. Since all industries are in the *same* country, the nominal exchange rate is 1/1 and the common deflator, $P_{0,t}$, means that the real exchange value of $r_t^j X_t^j$ between industries is $(1/P_{0,t})/(1/P_{0,t}) = 1$, implying PPP.

From above, the GEN framework yields a simple formula for an industry’s contribution to real GDP growth using PPP values. Letting $r_t^j X_t^j = X_t^{*j}$ where X_t^{*j} is in PPP value, (14) yields

$$X_{t-1} = \sum_j r_{t-1}^j X_{t-1}^j = \sum_j X_{t-1}^{*j} \quad ; \quad X_t = \sum_j r_t^j X_t^j = \sum_j X_t^{*j}. \quad (19)$$

Moreover, shares of GDP in PPP are the same as shares of nominal GDP as shown by

$$\frac{X_{t-1}^{*j}}{X_{t-1}} = \frac{r_{t-1}^j X_{t-1}^j}{X_{t-1}} = \frac{P_{0,t-1}^j X_{t-1}^j}{P_{0,t-1} X_{t-1}} = \frac{Y_{t-1}^j}{Y_{t-1}}. \quad (20)$$

It follows from (19) and (20) that

$$\frac{X_t}{X_{t-1}} - 1 = \sum_j \frac{X_{t-1}^{*j}}{X_{t-1}} \left(\frac{X_t^{*j}}{X_{t-1}^{*j}} - 1 \right) = \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} \left(\frac{X_t^{*j}}{X_{t-1}^{*j}} - 1 \right). \quad (21)$$

That is, the growth of real GDP in chained or in constant prices equals the weighted sum of the growth of each industry’s real GDP in PPP values where the weight is the industry’s share in the

⁹ The use of the PPP concept in (18) is unusual. To express (18) in the usual case of “consumer” PPP, the GDP deflators, P^S and P^K , need only to be replaced by the corresponding US and UK consumer price indexes.

economy's nominal GDP. Moreover, recalling PGE in (12) and PCE in (13), it can be verified that

$$\text{PGE} + \text{PCE} = \frac{Y_{t-1}^j}{Y_{t-1}^j} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right) + \frac{X_t^j}{X_{t-1}^j} (r_t^j - r_{t-1}^j) = \frac{Y_{t-1}^j}{Y_{t-1}^j} \left(\frac{X_t^{*j}}{X_{t-1}^{*j}} - 1 \right). \quad (22)$$

The right-hand side of (22) shows a direct formula for an industry's growth contribution using PPP values. This direct formula combines PGE (pure growth effect) and PCE (price change effect) shown in Table 2 for the US and in Table 4 for the Philippines. The result in (22) is confirmed for each industry by the equality of the results in the last columns of Table 2 and Table 5 for the US and also by the equality of the results in the last columns of Table 4 and Table 6 for the Philippines.

It is important to note that industry real GDP as presently computed – for example, US GDP in chained prices in columns (1) and (2) in Table 5 and Philippine GDP in constant prices in columns (1) and (2) in Table 6 – are the ones relevant for studying industries *individually* or in *isolation*. However, because these real GDPs differ in units of measure between industries, this paper argues that real GDP of industries in PPP values in columns (5) and (6) are the ones valid for determining industry contributions to the level and growth of the economy's real GDP in chained or in constant prices. The values in columns (5) and (6) are themselves the level contributions of industries while those in column (7) are their growth contributions. It may be noted that these level contributions as well as growth contribution *exactly add up* to the economy's real GDP level and growth, as shown by zero residuals. Without this additivity, residuals will arise and will put to question the “correctness” of the above contributions.

Finally, if separate quantity and relative price effects on growth are desired, the growth contributions in column (7) of Table 5 and Table 6 may be broken out into PGE (pure growth effect) and price change effect (PCE) as shown in Table 2 for the US and in Table 4 for the Philippines.

	BEA		GEN				
	GDP in chained Prices		Relative prices		GDP in PPP values		GDP growth
	(billion chained 2009 dollars)		(weights)		(billion chained 2009 dollars)		(percent)
	2013	2014	2013	2014	2013	2014	2014
(1)	(2)	(3)	(4)	(5)	(6)	(7)	
US GDP	15,583.3	15,961.7	1.00	1.00	15,583.3	15,961.7	2.43
Industry GDP weighted by relative prices					(1)x(3)	(2)x(4)	
Agriculture, forestry, fishing, and hunting	145.8	149.6	1.446	1.325	210.8	198.2	-0.081
Mining	335.3	358.7	1.230	1.164	412.4	417.5	0.033
Utilities	275.9	264.8	0.917	0.976	253.0	258.4	0.035
Construction	584.1	589.6	0.993	1.036	579.7	610.9	0.200
Durable goods	1,078.3	1,095.9	0.938	0.945	1,011.9	1,035.5	0.152
Nondurable goods	789.3	801.9	1.117	1.115	881.5	894.5	0.083
Wholesale trade	919.3	950.1	1.020	1.011	937.3	961.0	0.152
Retail trade	907.7	924.0	0.997	0.994	904.9	918.1	0.084
Transportation and warehousing	442.0	445.7	1.023	1.044	452.2	465.3	0.084
Information	795.4	826.2	0.933	0.918	742.4	758.8	0.105
Finance and insurance	994.1	1,016.7	1.082	1.107	1,075.7	1,125.2	0.318
Real estate and rental and leasing	2,052.0	2,100.7	0.978	0.984	2,006.3	2,068.1	0.396
Professional, scientific, and technical services	1,071.6	1,107.3	0.992	0.991	1,063.0	1,097.7	0.223
Management of companies and enterprises	313.7	335.3	0.960	0.927	301.1	310.9	0.063
Administrative and waste management services	483.0	503.8	0.956	0.961	461.8	484.0	0.142
Educational services	164.2	167.4	1.052	1.060	172.7	177.4	0.030
Health care and social assistance	1,117.5	1,141.6	0.995	0.989	1,111.5	1,128.8	0.111
Arts, entertainment, and recreation	157.2	161.8	0.977	0.980	153.7	158.6	0.032
Accommodation and food services	431.7	444.8	1.000	1.009	431.5	449.0	0.112
Other services, except government	327.5	335.6	1.037	1.046	339.6	351.1	0.074
Federal government	661.9	656.1	1.001	1.007	662.5	660.6	-0.012
State and local government	1,388.7	1,390.9	1.021	1.030	1,418.0	1,432.2	0.091
Residuals: "Not allocated by industry"	123.9	164.9			0	0	0

Source: Author's calculations of PPP level in (18) and growth in (22) applied to US GDP in Table 1.

	TRAD		GEN				
	GDP in constant prices		Relative prices		GDP in PPP values		GDP growth
	(million constant 2000 pesos)		(weights)		(million constant 2000 pesos)		(percent)
	2014	2015	2014	2015	2014	2015	2015
(1)	(2)	(3)	(4)	(5)	(6)	(7)	
Philippine GDP	7,164,016	7,579,941	1.000	1.000	7,164,016	7,579,941	5.81
Industry GDP weighted by relative prices					(1)x(3)	(2)x(4)	
Agriculture and forestry	587,329	591,215	1.188	1.127	697,545	666,568	-0.432
Fishing	130,495	128,109	0.856	0.871	111,706	111,631	-0.001
Mining and quarrying	76,474	75,444	0.929	0.785	71,052	59,238	-0.165
Manufacturing	1,666,514	1,762,103	0.885	0.864	1,475,357	1,523,162	0.667
Construction	422,150	459,586	1.112	1.134	469,278	521,350	0.727
Electricity gas and water supply	229,555	240,625	1.016	0.988	233,291	237,681	0.061
Transport communication and storage	536,562	579,054	0.827	0.842	443,966	487,401	0.606
Trade and repair of vehicles, personal, and household goods	1,184,994	1,266,656	1.073	1.082	1,271,151	1,370,343	1.385
Financial intermediation	515,484	545,076	1.087	1.110	560,358	605,056	0.624
Real estate renting and business activity	803,241	861,581	1.096	1.135	880,228	977,987	1.365
Public administration, defense, and social security	292,441	294,229	0.975	0.982	285,088	289,042	0.055
Other services	718,777	776,263	0.925	0.941	664,996	730,484	0.914
Residuals	0	0			0	0	0

Source: Author's calculations of PPP level in (18) and growth in (22) applied to Philippine GDP in Table 3.

Summary and conclusion

Real GDP of industries as presently computed are limited in use to studying industries *individually* or in *isolation* because they differ in units of measure due to different deflators. For this reason, they need relative prices as weights to convert them to the same units for valid comparative analysis in a *group* setting, as in this paper, in determining and comparing industry contributions to the level and growth of the economy's real GDP. Unfortunately, relative prices are ignored in existing procedures for real GDP in chained or in constant prices.

In light of the above, the present procedure of simple addition of industry GDP in constant prices to obtain the economy's GDP – while true – cannot be the rule because this paper's alternative GDP aggregation procedure is also valid in this case. Moreover, given that nominal GDP is additive and that economy-wide and individual industry GDP deflators are different, the present procedure involves addition of *different* commodity baskets akin to “adding apples and oranges” and, thus, objectionable. A further objection to the above simple addition is that it appears to violate additivity of nominal GDP except in the base period.

However, this paper showed that employing relative prices – ratios of industry GDP deflators to the economy's GDP deflator – as weights of industry GDP in constant prices will resolve the above concerns. Moreover, these relative price weights also apply to industry GDP in chained prices. With the above weights, this paper presented a general (GEN) GDP framework to determine the effects of differences and changes in relative prices on industry contributions to the level and growth of GDP in chained or in constant prices. Unless relative prices are constant, ignoring them will result in residuals in contributions to both the level and growth of GDP in chained prices. In the case of GDP in constant prices, ignoring them will not yield residuals but will result in the following economically misleading results that also apply to GDP in chained prices.

If relative prices are ignored, the level contributions of industries with above (below) average prices are understated (overstated) and growth contributions of industries with rising (falling) prices are understated (overstated). These results were borne out by US GDP in chained prices and Philippine GDP in constant prices. However, the above misleading results could be mitigated by this paper's GEN formulas for level and growth contributions that encompass existing formulas as special cases of constant relative prices.

In principle, relative prices are necessary for converting *different* industry real GDP to the *same* units (i.e., in PPP values) for additivity to equal the economy's real GDP. Industry GDP in PPP value is the industry's contribution to the *level* of the economy's real GDP. Using PPP values, this paper illustrated a direct formula for industry contributions to the *growth* of the economy's real GDP that *combines* the effects of changes in quantities and of changes in relative prices. However, this paper's GEN framework also illustrated a formula that *separates* growth contributions of industries into PGE (pure growth effect) for changes in quantities and PCE (price change effect) for changes in relative prices where the sum of PGE and PCE equals the above combined effects when using PPP values.

In sum, the GEN framework in this paper employs relative prices to convert industry real GDP in chained or in constant prices into the same or homogeneous units so that the level and growth contributions of industries correspondingly add up exactly to the "actual" level and growth of the economy's real GDP. Without relative prices, existing procedures for the above industry contributions are questionable.

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