# Consistency in Aggregation of GDP Indexes and Uniqueness of Quantity and Price Effects on Growth of GDP and Aggregate Labor Productivity

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Abstract

In traditional decomposition of GDP growth in constant prices, an industry's contribution consisted only of a quantity effect from GDP growth. Tang and Wang's (2004, 2014) innovation added a price effect from relative price change. Dumagan (2013a, 2016) showed that Tang and Wang's quantity and price effects for all industries exactly add up to growth of GDP either in chained or in constant prices, that is, regardless of the GDP index. However, this paper shows that it is only when GDP is in chained prices and the GDP index is consistent-in-aggregation (CIA) that quantity and price effects are invariant with industry regroupings, that is, unique. Therefore, Tang and Wang's (2004, 2014) growth decompositions in Canada and US—where GDP is in chained prices based on the Fisher index—yield effects that vary with industry regroupings because the Fisher index is not CIA. This variation prevents attributing unique price and quantity effects to industries and, thus, clouds Tang and Wang's analysis of the role of industries in GDP growth and in aggregate labor productivity growth. This paper also examines price and quantity effects on GDP growth of representative countries with GDP different from that in the US to make the results globally relevant.

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## Introduction

Industry contributions to GDP growth were limited to quantity effects until Tang and Wang (2004) introduced relative price–ratio of an industry GDP deflator to the economy's GDP deflator—as a weight of each industry's real GDP such that the weighted sum equals the economy's real GDP. Consequently, Tang and Wang (2004, 2014) showed that contributions to GDP growth consist of a *quantity* effect due to growth of the industry's real GDP and a *price* effect due to a change in the industry's relative price. Dumagan (2013a, 2016) showed that these effects are exactly additive in that their sum equals actual growth of GDP either in chained or in constant prices, that is, regardless of the index formula underlying GDP. For this reason, Dumagan (2013a) called Tang and Wang's growth decomposition a "generalized exactly additive decomposition" (GEAD) originally to describe their (Tang & Wang, 2004) decomposition of aggregate labor productivity (ALP) growth. Thus, henceforth in this paper, GEAD generally refers to their growth decomposition of ALP or GDP.

This paper points out that *uniqueness* of quantity and price effects is an analytic issue with important practical implications that has not been examined in GEAD. Uniqueness is important because it permits unambiguous attributions of values of the above effects to specific industries, which would not be possible otherwise. In cases without uniqueness, there is ambiguity in the magnitude of quantity and price effects that translates to incorrect inferences and possibly leads to misguided growth policies.

It is shown that price effects—which could be positive, zero, or negative—are unique *if they sum to zero* for all industries and, in this case, quantity effects are also unique since the sum of price and quantity effects of an industry remains the *same* even with regrouping of industries. The intuition behind the above condition is that relative price is the ratio of an industry's GDP deflator to the economy's GDP deflator. Considering that the economy's deflator is like the average of industry deflators, relative price may rise for some industries; remain the same; or fall for others. Hence, the sum of price effects will tend to zero. It is shown that in GEAD this sum necessarily equals zero if GDP is in chained prices and the underlying GDP indexes are consistent-in-aggregation (CIA). In this case, the sum of quantity effects equals GDP growth because GDP growth equals the overall sum of price and quantity effects.

In an application to Italian GDP in chained prices based on Laspeyres quantity and Paasche price indexes (European Union, 2007), the sum of price effects equals zero and the sum of quantity effects equals GDP growth. Price and quantity effects are unique in the sense that they are invariant with regrouping of existing industries, reflecting the fact that the above indexes are CIA (Vartia, 1976; Diewert, 1978).

In contrast, in an application to US GDP in chained prices based on Fisher quantity and price indexes, the sum of price effects does not equal to zero so that the sum of quantity effects does not equal to GDP growth, although their overall sum equals GDP growth. Moreover, statistical tests reject the hypothesis that the sum of price effects equals to zero in US GDP. Hence, price and quantity effects are not unique by varying with industry regrouping, resulting from the fact that Fisher indexes are not CIA (Diewert, 1978). These findings appear ironic in that GEAD was first applied by Tang and Wang (2004) and recently (2014) to GDP in chained prices based on Fisher indexes in Canada (Chevalier, 2003) and US (Landefeld & Parker, 1997) where they focused especially on price effects on growth of GDP and ALP.

Similarly, in an application to Philippine GDP in constant prices based on direct Laspeyres quantity and Paasche price indexes, the sum of price effects does not equal to zero so that the sum of quantity effects does not equal GDP growth, although their overall sum equals

GDP growth. However, in contrast to the case in the US, statistical tests cannot reject the hypothesis that the sum of price effects equals zero in Philippine GDP. Hence, price and quantity effects are *statistically* unique, that is, practically invariant with industry regrouping. This finding does not appear surprising because the *direct* Laspeyres quantity index—underlying GDP in constant prices—is also CIA (Balk, 2010), although GEAD requires the Laspeyres quantity index underlying GDP in chained prices for *analytically* unique price and quantity effects.

The above statistical test findings may not be generalizable to countries with GDP in chained prices like that in the US (e.g., Canada) or to countries other than the Philippines with GDP in constant prices. At the very least, however, they underscore the importance of testing the hypothesis that the sum of price effects equals zero prior to attempting interpretations of price and quantity effects in industry contributions to GDP growth.

The rest of this paper is organized as follows: Section 2 presents the GEAD decomposition of GDP growth; derives the formulas for quantity and price effects; and shows that uniqueness of the above effects requires GDP in chained prices and CIA of the GDP index. Section 3 illustrates the analytic results with Italian, US, and Philippine GDP—representing current official GDP practices in all countries—to make the framework practical and empirical findings relevant globally. Section 4 explores the implications of GDP growth decomposition results on Tang and Wang's decomposition of ALP growth. Section 5 concludes this paper.

## GEAD Framework for GDP Growth Decomposition

### Quantity and Price Effects on GDP Growth

Published national accounts in period $t$, for example, quarter or year, provide values of nominal GDP, $Y_t$ and $Y_t^j$, and the corresponding real GDP, $X_t$ and $X_t^j$, where $Y_t$ and $X_t$ represent the economy's GDP while $Y_t^j$ and $X_t^j$ represent GDP of a sector or industry denoted by $j$.

By definition, $X_t$ is obtained by dividing $Y_t$ by the economy's GDP price index or deflator, $P_{0,t}$, so that $X_t$ is valued in prices of the base year denoted by 0. Industry real GDP, $X_t^j$, is similarly obtained from industry nominal GDP, $Y_t^j$, using the industry's GDP deflator, $P_{0,t}^j$. From these definitions, the deflators $P_{0,t}$ and $P_{0,t}^j$ may be obtained implicitly by

$$P_{0,t} \equiv \frac{Y_t}{X_t} \quad ; \quad P_{0,t}^j \equiv \frac{Y_t^j}{X_t^j} . \tag{1}$$

By property, nominal GDP is additive. That is,

$$Y_t = \sum_j Y_t^j . \tag{2}$$

In contrast, real GDP is not necessarily additive. If the deflators $P_{0,t}$ and $P_{0,t}^j$ are direct Paasche price indexes, then real GDP is in constant prices that is additive, that is, $X_t = \sum_j X_t^j$. On the other hand, if the deflators are chained Paasche or Fisher price indexes, then real GDP is in chained prices that is not additive, that is, $X_t \neq \sum_j X_t^j$, except in the base year (Balk, 2010).

It follows from (1) and (2) that

$$P_{0,t}\, X_t = \sum_j P_{0,t}^j X_t^j \quad ; \quad r_t^j \equiv \frac{P_{0,t}^j}{P_{0,t}} \quad ; \quad X_t = \sum_j r_t^j X_t^j = \sum_j X_t^{*j} . \tag{3}$$

Tang and Wang's (2004, 2014) framework employs (3) that Dumagan (2013a) pointed out applies to GDP in chained or in constant prices, that is, regardless of the index formulas underlying the deflators, $P_{0,t}$ and $P_{0,t}^j$. Therefore, (3) is perfectly general so that it applies to any real GDP.

In (3), $r_t^j \equiv P_{0,t}^j / P_{0,t}$ is a relative price by being a ratio of an industry's GDP deflator to the economy's GDP deflator. As such, $r_t^j$ serves as a real price that converts each industry's real GDP, $X_t^j$, to $X_t^{*j}$ in the same unit as the economy's real GDP. Therefore, $X_t^{*j}$ is additive across industries even if $X_t^j$ is not be additive.

It follows from (3) that the relative change in real GDP is

$$\frac{X_t}{X_{t-1}} = \sum_j r_t^j \, w_{t-1}^j \frac{X_t^j}{X_{t-1}^j} \quad ; \quad X_{t-1} = \sum_j r_{t-1}^j X_{t-1}^j \quad ; \quad w_{t-1}^j \equiv \frac{X_{t-1}^j}{X_{t-1}} \, . \tag{4}$$

It may be recognized that $X_t / X_{t-1}$ is the implicit aggregate GDP quantity index while $X_t^j / X_{t-1}^j$ is the corresponding implicit industry GDP quantity index. Thus, (4) states that the implicit aggregate GDP quantity index equals the weighted sum of implicit industry GDP quantity indexes where the industry weights are given by $r_t^j w_{t-1}^j$ that do not necessarily sum to 1 unless relative prices are constant, that is, $r_{t-1}^j = r_t^j$. Moreover, note that $w_{t-1}^j$ does not necessarily sum to 1 if GDP is not in constant prices. However, (1) to (4) yield

$$r_{t-1}^j w_{t-1}^j = \left(\frac{Y_{t-1}^j / X_{t-1}^j}{Y_{t-1} / X_{t-1}}\right)\left(\frac{X_{t-1}^j}{X_{t-1}}\right) = \frac{Y_{t-1}^j}{Y_{t-1}} \quad ; \quad \sum_j r_{t-1}^j w_{t-1}^j = \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} = 1 \, . \tag{5}$$

From all the above, it can be verified that

$$\frac{X_t}{X_{t-1}} - 1 = \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1\right) + \sum_j \frac{X_t^j}{X_{t-1}} \left(r_t^j - r_{t-1}^j\right) . \tag{6}$$

This result is the GEAD formula for the growth rate of the economy's GDP in chained or in constant prices. Since it is true for any real GDP, (6) equals the "actual" GDP growth rate.

**CIA of GDP Indexes and Uniqueness of Quantity and Price Effects**

In (6), GDP growth comes from two sources. One is PGE—corresponding to Tang and Wang's quantity effect from growth in the industry's real GDP—defined by

$$\text{PGE (pure growth effect)} \equiv \frac{Y_{t-1}^{j}}{Y_{t-1}}\left(\frac{X_t^j}{X_{t-1}^j} - 1\right). \tag{7}$$

The other is PCE—corresponding to Tang and Wang's price effect from a change in the industry's relative price—given by[1]

$$\text{PCE (price change effect)} \equiv \frac{X_t^j}{X_{t-1}}\left(r_t^j - r_{t-1}^j\right). \tag{8}$$

While PGE and PCE exactly add up to the actual growth of GDP either in chained or in constant prices, their values may vary with regrouping of existing industries depending on the underlying GDP indexes. This variation prevents attributing unique PGE and PCE values to specific industries and, thus, clouds analysis of the role of industries in GDP growth. To shed light on this issue of uniqueness, first compute the Sum of PCE in (8), using (3) to (5), to obtain

$$\text{Sum of PCE} \equiv \sum\nolimits_j \frac{X_t^j}{X_{t-1}}\left(r_t^j - r_{t-1}^j\right) = \frac{X_t}{X_{t-1}} - \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}}\frac{X_t^j}{X_{t-1}^j} \quad ; \quad \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}} = 1\,. \tag{9}$$

Note in (9) that $X_t/X_{t-1}$ is the economy's implicit GDP quantity index and $X_t^j/X_{t-1}^j$ is the implicit GDP quantity index of an industry. Depending on the index formula underlying GDP, CIA (Vartia, 1976; Diewert, 1978) means that $X_t/X_{t-1}$ equals the weighted sum of $X_t^j/X_{t-1}^j$ where the weights sum to 1. Therefore, if the GDP quantity index is CIA, (6) to (9) yield

$$\text{Sum of PCE} = 0 \quad ; \quad \frac{X_t}{X_{t-1}} = \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}}\frac{X_t^j}{X_{t-1}^j} \quad ; \quad \text{GDP growth} = \text{Sum of PGE}\,. \tag{10}$$

While PCE could be positive, zero, or negative for an individual industry, (9) and (10) show that PCE cancels out in the aggregate under CIA. In this case, there are no growth effects of relative

[1] Tang and Wang's (2014) notation may be different but (7) and (8) above can be derived from their expression (3), p. 7, for an industry's contribution to GDP growth. Moreover, the names and mathematical expressions for PGE in (7) and PCE in (8) above are the same as those in Dumagan (2016).

price changes that remain “unallocated” to an industry, implying that each industry’s PCE is unique. Therefore, the only other component, PGE, is also unique.

Moreover, suppose some individual industries are combined into a group. In this case, by implication of CIA, the values of PCE and PGE remain unique. Specifically, when the group is treated as one whole, the group PCE equals the sum of the individual PCE of the member industries. Also, the group PGE equals the sum of the individual PGE of the members.

To see the connection between Sum of PCE $= 0$ and CIA in practice, let $Y_0$ and $Y_0^j$ be the economy’s and an industry’s nominal GDP in the base year 0. Also, let $Q_{0,t-1}$ and $Q_{0,t}$ be the economy’s GDP quantity indexes in years 1 and 2 with the same base year; and let $Q_{0,t-1}^j$ and $Q_{0,t}^j$ be an industry’s corresponding quantity indexes. By definition of real GDP,[2]

$$X_{t-1} \equiv Y_0 Q_{0,t-1} \quad ; \quad X_t \equiv Y_0 Q_{0,t} \quad ; \quad X_{t-1}^j \equiv Y_0^j Q_{0,t-1}^j \quad ; \quad X_t^j \equiv Y_0^j Q_{0,t}^j \,. \tag{11}$$

It follows from (11) that (9) becomes

$$\text{Sum of PCE} \equiv \frac{Q_{0,t}}{Q_{0,t-1}} - \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} \left( \frac{Q_{0,t}^j}{Q_{0,t-1}^j} \right). \tag{12}$$

The expression in (12) applies generally. However, depending on the specific index formula, (12) may or may not equal to zero as shown below.

Suppose in period $t$ that nominal prices are $p_{it}^j$ and quantities are $q_{it}^j$ of $i = 1, 2, \cdots, N$ final commodities (i.e., goods and services) produced by $j = 1, 2, \cdots, M$ industries. GDP in current prices or nominal GDP is denoted by $Y_t^j$ for industry $j$ and by $Y_t$ for the entire economy. By definition, noting that nominal GDP is additive in any period, it is true that

[2] Real GDP may be computed either by *deflating* nominal GDP by a price index or inflating base-year nominal GDP by a quantity index. For example, in year $t$, the economy’s real GDP is $X_t \equiv Y_t / P_{0,t} = Y_0 Q_{0,t}$ and an industry’s real GDP is $X_t^j \equiv Y_t^j / P_{0,t}^j = Y_0^j Q_{0,t}^j$. These results follow from value index decomposition, that is, $Y_t / Y_0 = P_{0,t} Q_{0,t}$ or $Y_t^j / Y_0^j = P_{0,t}^j Q_{0,t}^j$, which is satisfied by the pair of Paasche price and Laspeyres quantity indexes or by the pair of Fisher price and quantity indexes (Fisher, 1922).

$$Y_{t-1}^{j} \equiv \sum_{i} p_{it-1}^{j} q_{it-1}^{j} \quad ; \quad Y_{t-1} \equiv \sum_{j} Y_{t-1}^{j} \quad ; \quad Y_{t}^{j} \equiv \sum_{i} p_{it}^{j} q_{it}^{j} \quad ; \quad Y_{t} \equiv \sum_{j} Y_{t}^{j} . \qquad (13)$$

In the case of chained indexes (Balk, 2010), the GDP quantity index is defined by

$$Q_{0,t-1} \equiv Q_{0,1} \times Q_{1,2} \times \cdots \times Q_{t-2,t-1} \quad ; \quad Q_{0,t} = Q_{0,t-1} \times Q_{t-1,t} . \qquad (14)$$

The definition in (14) applies analogously to industry chained quantity indexes. Therefore, for GDP in chained prices based on chained Laspeyres quantity indexes, (11), (13), and (14) yield

$$\frac{Q_{0,t}^{j}}{Q_{0,t-1}^{j}} = Q_{t-1,t}^{j} \equiv \frac{\sum_i p_{it-1}^{j} q_{it}^{j}}{\sum_i p_{it-1}^{j} q_{it-1}^{j}} \quad ; \quad \frac{Q_{0,t}}{Q_{0,t-1}} = Q_{t-1,t} \equiv \frac{\sum_j \sum_i p_{it-1}^{j} q_{it}^{j}}{\sum_j \sum_i p_{it-1}^{j} q_{it-1}^{j}} . \qquad (15)$$

Substituting (13) and (15) into (12) yields

$$\text{Sum of PCE} \equiv \frac{\sum_j \sum_i p_{it-1}^{j} q_{it}^{j}}{\sum_j \sum_i p_{it-1}^{j} q_{it-1}^{j}} - \sum_{j} \left( \frac{\sum_i p_{it-1}^{j} q_{it-1}^{j}}{\sum_j \sum_i p_{it-1}^{j} q_{it-1}^{j}} \right) \left( \frac{\sum_i p_{it-1}^{j} q_{it}^{j}}{\sum_i p_{it-1}^{j} q_{it-1}^{j}} \right) = 0. \qquad (16)$$

That is, GDP in chained prices based on the Laspeyres quantity index satisfies (16) and yields GDP growth $=$ Sum of PGE in (10) because this index is CIA (Vartia, 1976; Diewert, 1978). In contrast, GDP in chained prices based on the Fisher index yields Sum of PCE $\neq 0$ and GDP growth $\neq$ Sum of PGE because this index is not CIA (Diewert, 1978).

For GDP in constant prices, the indexes in (12) are direct Laspeyres quantity indexes (Balk, 2010) defined by

$$Q_{0,t-1}^{j} = \frac{\sum_i p_{i0}^{j} q_{it-1}^{j}}{\sum_i p_{i0}^{j} q_{i0}^{j}} \quad ; \quad Q_{0,t-1} = \frac{\sum_j \sum_i p_{i0}^{j} q_{it-1}^{j}}{\sum_j \sum_i p_{i0}^{j} q_{i0}^{j}} \; ; \qquad (17)$$

$$Q_{0,t}^{j} = \frac{\sum_i p_{i0}^{j} q_{it}^{j}}{\sum_i p_{i0}^{j} q_{i0}^{j}} \quad ; \quad Q_{0,t} = \frac{\sum_j \sum_i p_{i0}^{j} q_{it}^{j}}{\sum_j \sum_i p_{i0}^{j} q_{i0}^{j}} . \qquad (18)$$

Combining (12), (13), (17), and (18) yields

$$\text{Sum of PCE} \equiv \frac{\sum_j \sum_i p_{i0}^{j} q_{it}^{j}}{\sum_j \sum_i p_{i0}^{j} q_{it-1}^{j}} - \sum_{j} \left( \frac{\sum_i p_{it-1}^{j} q_{it-1}^{j}}{\sum_j \sum_i p_{it-1}^{j} q_{it-1}^{j}} \right) \left( \frac{\sum_i p_{i0}^{j} q_{it}^{j}}{\sum_i p_{i0}^{j} q_{it-1}^{j}} \right) \neq 0 . \qquad (19)$$

Therefore, from (10) and (19), GDP in constant prices based on the direct Laspeyres quantity index yields Sum of PCE $\neq 0$ and GDP growth $\neq$ Sum of PGE. This index is also CIA but this property requires shares of GDP in constant prices, $\sum_i p_{i0}^j q_{it-1}^j / \sum_j \sum_i p_{i0}^j q_{it-1}^j$, in place of shares of GDP in current prices, $\sum_i p_{it-1}^j q_{it-1}^j / \sum_j \sum_i p_{it-1}^j q_{it-1}^j$. It appears from (16) and (19) that GEAD admits CIA only of the Laspeyres quantity index underlying GDP in chained prices.

In summary, Sum of PCE $= 0$ and GDP growth $=$ Sum of PGE for GDP in chained prices based on the Laspeyres quantity index. In this case, there are no price effects that are unallocated to industries so that PCE and PGE are unique, unaffected by industry grouping. That is, group PCE (or PGE) equals the sum of the PCE (or PGE) of the group members. In contrast, Sum of PCE $\neq 0$ and GDP growth $\neq$ Sum of PGE for GDP in chained prices based on the Fisher index and also for GDP in constant prices based on a direct Laspeyres quantity index.

However, the next section presents statistical test results—using annual data (1997-2015) on US GDP in chained prices based on the Fisher index—showing that the null hypothesis (Sum of PCE $= 0$) can be rejected, that is, Sum of PCE $\neq 0$ is statistically significant in US GDP. In contrast, a similar statistical test—using quarterly data (2008-2015) on Philippine GDP in constant prices based on the direct Laspeyres quantity index—shows that the null hypothesis cannot be rejected, that is, Sum of PCE $= 0$ is statistically significant in Philippine GDP. Therefore, for practical purposes, PGE and PCE values in US GDP can be treated as varying with industry grouping so that there is ambiguity in attributing unique values to specific industries while they can be treated as unique and there is no such ambiguity in Philippine GDP.

**Illustrations of GEAD Contributions to GDP Growth**

In all illustrations, GDP growth is computed over the years 2009 to 2010, which were chosen for the following reason. Consider that GDP in chained prices is not additive, that is, it is not necessarily equal to the simple sum of the GDP in chained prices of industries, except in the base year.[3] Hence, the difference between the economy's GDP and the above sum is a residual, which is necessarily zero only in the base year. It happens that 2000 is the base year in Italy while 2009 is the base year in the US. Hence, the residual is non-zero in Italy for both years 2009 and 2010 (Table 1) while the residual is zero in 2009 and non-zero in 2010 in the US (Table 3). Thus, it appears that the situation is disadvantageous to Italy for having two non-zero residuals compared to the US in having only one non-zero residual. The years 2009 and 2010 were chosen to show that the above disadvantage is immaterial because the desired analytic results—that is, *exact additivity* and uniqueness of PGE and PCE—both hold for Italy (Table 2) but only exact additivity holds for the US (Table 4), as shown later.

In the case of the Philippines, there are zero residuals in 2009 and 2010 (Table 5) because GDP in constant prices equals the simple sum of GDP in constant prices of industries. In the Philippines, PGE and PCE of industry growth contributions are exactly additive but not unique as they appear (Table 6), although as noted earlier statistical tests show the null hypothesis, Sum of PCE $= 0$, cannot be rejected, implying that for practical purposes PGE and PCE are unique and attributable to industries in Philippine GDP.

**Exactly Additive and Unique Quantity and Price Effects in Italian GDP**

From the preceding analysis, Sum of PCE = 0 implies unique PGE and PCE in industry contributions to GDP growth. This condition is satisfied by GDP in chained prices based on Laspeyres quantity and Paasche price indexes in Italy (Table 1) and in other EU countries.[4]

[3] "Non-additivity" holds in all countries with GDP in chained prices. For some country practices, see Chevalier (2003) for Canada; Landefeld and Parker (1997) for the US; Schreyer (2004) and EU (2007) for EU and OECD countries.

[4] Brueton (1999) noted that the EU System of National Accounts 1995 recommended Laspeyres quantity and Paasche price indexes as more practical than the theoretically superior Fisher quantity and price indexes adopted by Canada and the US as recommended by the UN System of National Accounts 1993.

| Table 1. Value-Added and Employment in Italy, 2009-2010 | | | | | | |
|---|---|---|---|---|---|---|
| | **Value-added in in current prices** (millions of euros) | | **Value-added in chained prices** (millions of 2000 euros) | | **Employment in persons** (thousands) | |
| | **2009** | **2010** | **2009** | **2010** | **2009** | **2010** |
| Agriculture, hunting and forestry; fishing and operation of fish hatcheries and fish farms | 25,886 | 26,370 | 28,379 | 28,665 | 967 | 983 |
| Industry, including energy | 260,237 | 268,437 | 208,201 | 218,251 | 4,970 | 4,787 |
| Construction | 84,819 | 82,761 | 55,949 | 54,023 | 1,935 | 1,907 |
| Wholesale and retail trade, repair of motor vehicles and household goods, hotels and restaurants; transport and communication | 304,350 | 307,514 | 253,973 | 260,836 | 6,057 | 6,024 |
| Financial, real estate, renting and business activities | 389,123 | 393,613 | 293,776 | 295,588 | 3,694 | 3,716 |
| Other service activities (as one whole) | 303,267 | 308,248 | 233,164 | 232,968 | 7,217 | 7,241 |
| Public administration and defence; compulsory social security | 93,644 | 94,962 | 68,573 | 68,281 | 1,342 | 1,333 |
| Education | 67,371 | 66,656 | 54,477 | 54,532 | 1,583 | 1,560 |
| Health and social work | 83,409 | 86,481 | 67,080 | 67,304 | 1,650 | 1,667 |
| Other community, social and personal service activities | 43,518 | 44,508 | 31,350 | 31,266 | 1,123 | 1,128 |
| Private households with employed persons | 15,325 | 15,640 | 11,811 | 11,745 | 1,520 | 1,554 |
| **Value-added and employment** | **1,367,681** | **1,386,942** | **1,076,071** | **1,092,021** | **24,839** | **24,658** |
| **Residual** | **0.0** | **0.0** | **2,503** | **1,531** | **0.0** | **0.0** |
| **Source**: Istat - Istituto Nazionale di Statistica. | | | | | | |

As noted earlier, GDP or value-added in chained prices in Italy is not additive, as shown by residuals (Table 1). These residuals are, however, immaterial in the GEAD framework to the exact additivity and uniqueness of PGE and PCE. Table 2 shows exact additivity in the last column where the sum 1.4822 equals the actual GDP growth. CIA is confirmed by the result that Sum of PCE = 0. In turn, this implies uniqueness as explained below.

Row 6 (Table 2) shows the contribution of "Other services activities (as one whole)" when the five individual service categories (Table 1) are treated as one whole group. In comparison, row 7 (Table 2) shows the sum of the contributions of these five individual service categories. CIA implies that the contribution of the group taken as one whole equals the sum of the individual contributions of the members of the group. Moreover, this equality applies to the PGE and PCE components. As shown, the PGE contribution of −0.0186 percentage points and PCE contribution of 0.3992 percentage points of the group as one whole (row 6) equals the corresponding sums of the individual PGE and PCE contributions of the group members (row 7). The invariance above of PGE and PCE with regrouping implies that the individual PGE and PCE of the members of the group are each invariant, that is, unique. Moreover, since the regrouping involves only the service industries in row 6, there should be no change in the PGE and PCE components of the growth contributions of the non-service industries. Thus, the PGE and PCE results under CIA are all unique.

| | **Table 2. Value-Added Growth in Italy, 2009-2010** | | | |
|---|---|---|---|---|
| | | **PGE** | **PCE** | **PGE + PCE** |
| | GEAD growth contributions by industry | (Percentage points) | | |
| 1 | Agriculture, hunting and forestry; fishing and operation of fish hatcheries and fish farms | 0.0191 | 0.0177 | 0.0368 |
| 2 | Industry, including energy | 0.9185 | -0.3046 | 0.6139 |
| 3 | Construction | -0.2136 | 0.0675 | -0.1461 |
| 4 | Wholesale and retail trade, repair of motor vehicles and household goods, hotels and restaurants; transport and communication | 0.6013 | -0.3535 | 0.2478 |
| 5 | Financial, real estate, renting and business activities | 0.1756 | 0.1738 | 0.3493 |
| **6** | **Other service activities (as one whole)** | **-0.0186** | **0.3992** | **0.3806** |
| **7** | **Other service activities (by category)** | **-0.0186** | **0.3992** | **0.3806** |
| **8** | **Sum (1 to 5, and 6)** | **1.4822** | **0.0000** | **1.4822** |
| **9** | **Sum (1 to 5, and 7)** | **1.4822** | **0.0000** | **1.4822** |
| **10** | **Value-added growth** | | | **1.4822** |
| | **Source**: Author's calculations of PGE for the *quantity* effect in (7) and PCE for the *price* effect in (8) from data in Table 1. | | | |

Uniqueness is important because it permits the assertion that (i) the contribution to Italy's GDP growth in 2010 of "Other services activities," for example, was 0.3806 percentage points (Table 2, row 6 or row 7) and that (ii) this consisted of −0.0186 percentage points from PGE and 0.3992 percentage points from PGE. This applies similarly to each of the other industries. It is important to note that uniqueness permits asserting both (i) and (ii). Without uniqueness, however, (i) can be asserted but not (ii).

Finally, in Table 2, CIA implies GDP growth = Sum of PGE = 1.4822% and Sum of PCE = 0 in 2010. These results that GDP growth = Sum of PGE and Sum of PCE = 0 have analytic basis and can be verified to be true in Italian GDP in other years. However, the result that Sum of PCE = 0 for the whole economy does not imply that the PCE of individual industries may be ignored. The reason is that PCE is the growth contribution of an industry from a change in the industry's relative price or real exchange value of its output relative to the output of other industries. Therefore, ignoring PCE simply because Sum of PCE = 0 would misrepresent the growth contribution of an industry.[5]

**Exactly Additive but not Unique Quantity and Price Effects in US GDP**

US GDP in chained prices (Table 3) has 2009 as the base year. By definition, GDP in current prices equals GDP in chained prices and both are additive in the base year, as shown by the zero residual in 2009. In other years, GDP in chained prices is not additive, as shown by the non-zero 2010 residual.

**Table 3. Gross Domestic Product and Employment in the United States, 2009-2010**

| | GDP in current prices (billions of dollars) | | GDP in chained prices (billions of 2009 dollars) | | Employment (thousands) | |
|---|---|---|---|---|---|---|
| | **2009** | **2010** | **2009** | **2010** | **2009** | **2010** |
| Agriculture, forestry, fishing, and hunting | 137.7 | 160.2 | 137.7 | 140.3 | 1907 | 1935 |
| Mining | 290.3 | 331.7 | 290.3 | 272.7 | 648 | 662 |
| Utilities | 250.8 | 267.0 | 250.8 | 274.4 | 556 | 546 |
| Construction | 577.3 | 541.6 | 577.3 | 551.6 | 7,657 | 7,189 |
| Manufacturing | 1,726.7 | 1,830.6 | 1,726.7 | 1,818.2 | 11,849 | 11,538 |
| Wholesale trade | 822.8 | 868.5 | 822.8 | 848.3 | 5,581 | 5,476 |
| Retail trade | 842.1 | 868.8 | 842.1 | 862.1 | 13,500 | 13,358 |
| Transportation and warehousing | 398.8 | 425.1 | 398.8 | 421.4 | 4,414 | 4,338 |
| Information | 705.3 | 730.2 | 705.3 | 735.1 | 2,775 | 2,658 |
| Professional and business services | 1,661.1 | 1,729.7 | 1,661.1 | 1,718.0 | 17,613 | 17,860 |
| Educational services, health care, and social assistance | 1,214.0 | 1,248.5 | 1,214.0 | 1,220.5 | 18,590 | 18,880 |
| Arts, entertainment, recreation, accommodation, and food services | 522.3 | 540.7 | 522.3 | 541.3 | 11,181 | 11,150 |
| Other services, except government | 329.5 | 332.4 | 329.5 | 323.9 | 6,820 | 6,684 |
| Government | 2,065.8 | 2,137.9 | 2,065.8 | 2,079.8 | 20,506 | 20,441 |
| Finance, insurance, real estate, rental, and leasing (as one whole) | 2,874.1 | 2,951.6 | 2,874.1 | 2,925.4 | 8,112 | 8,001 |
| Federal Reserve banks, credit intermediation, and related activities | 399.5 | 410.2 | 399.5 | 388.3 | 2,562 | 2,507 |
| Securities, commodity contracts, and investments | 186.7 | 199.5 | 186.7 | 192.2 | 893 | 869 |
| Insurance carriers and related activities | 357.6 | 365.2 | 357.6 | 359.7 | 2,300 | 2,313 |
| Funds, trusts, and other financial vehicles | 25.5 | 31.0 | 25.5 | 28.6 | 82 | 82 |
| Real estate | 1,740.6 | 1,783.9 | 1,740.6 | 1,794.8 | 1,733 | 1,726 |
| Rental and leasing services and lessors of intangible assets | 164.2 | 161.8 | 164.2 | 162.4 | 542 | 504 |
| **Gross domestic product** | **14,418.6** | **14,964.5** | **14,418.6** | **14,783.8** | **131,709** | **130,716** |
| **Residual** | **0.0** | **0.0** | **0.0** | **38.7** | **0.0** | **0.0** |

**Source:** Bureau of Economic Analysis.

[5] A major reason for the shift in measuring real GDP from constant to chained prices is to correctly account for the effects of relative price changes on GDP growth by avoiding overestimation (underestimation) of the growth contributions of GDP components whose prices on average have fallen (risen) since the base period. In the US, a major motivation for the shift to chained prices was to correct for the overestimation of the growth contribution of information technology products whose prices were falling rapidly (Landefeld & Parker, 1997).

Table 4 presents the GEAD decomposition of US GDP growth. Exact additivity is shown in the last column where the sum of PGE + PCE for all industries equals the actual GDP growth of 2.5328%. The lack of CIA is confirmed by Sum of PCE ≠ 0 and GDP growth ≠ Sum of PGE in row 15 or 16. In turn, these results imply lack of uniqueness of PGE and PCE as illustrated in Table 4.

**Table 4. GDP Growth in the United States, 2009-2010**

| | | PGE | PCE | PGE + PCE |
|---|---|---|---|---|
| | GEAD growth contributions by industry | (Percentage points) | | |
| 1 | Agriculture, forestry, fishing, and hunting | 0.0180 | 0.1246 | 0.1426 |
| 2 | Mining | -0.1221 | 0.3814 | 0.2594 |
| 3 | Utilities | 0.1637 | -0.0737 | 0.0900 |
| 4 | Construction | -0.1782 | -0.1147 | -0.2930 |
| 5 | Manufacturing | 0.6346 | -0.0673 | 0.5673 |
| 6 | Wholesale trade | 0.1769 | 0.0674 | 0.2442 |
| 7 | Retail trade | 0.1387 | -0.0263 | 0.1124 |
| 8 | Transportation and warehousing | 0.1567 | -0.0099 | 0.1468 |
| 9 | Information | 0.2067 | -0.0951 | 0.1115 |
| 10 | Professional and business services | 0.3946 | -0.0637 | 0.3309 |
| 11 | Educational services, health care, and social assistance | 0.0451 | 0.0896 | 0.1347 |
| 12 | Arts, entertainment, recreation, accommodation, and food services | 0.1318 | -0.0494 | 0.0823 |
| 13 | Other services, except government | -0.0388 | 0.0311 | -0.0077 |
| 14 | Government | 0.0971 | 0.2239 | 0.3210 |
| **15** | **Finance, insurance, real estate, rental, and leasing (as one whole)** | **0.3565** | **-0.0662** | **0.2903** |
| **16** | **Finance, insurance, real estate, rental, and leasing (by category)** | **0.3600** | **-0.0696** | **0.2903** |
| **17** | **Sum (1 to 14, and 15)** | **2.1812** | **0.3516** | **2.5328** |
| **18** | **Sum (1 to 14, and 16)** | **2.1847** | **0.3482** | **2.5328** |
| **19** | **GDP growth** | | | **2.5328** |

**Source**: Author's calculations of PGE for the *quantity* effect in (7) and PCE for the *price* effect in (8) from data in Table 3.

Row 15 shows the contribution of "Finance, insurance, real estate, rental, and leasing (as one whole)" when the six financial service industries (Table 3) are treated as one whole group. In comparison, row 16 shows the sum of the individual contributions of these six industries. Lack of CIA is shown by the differences in rows 15 and 16. Notice that the PGE contribution of 0.3565 percentage points of the group (row 15) differs from the sum of the PGE contributions of 0.3600 percentage points of the group members (row 16). Similarly, the PCE contribution of −0.0662 percentage points of the group differs from the sum of the PCE contributions of

−0.0696 percentage points of the group members. However, while PGE and PCE change with regrouping, their sum remains the same equal to 0.2903.

Thus, in the above example, lack of uniqueness permits the assertion that the growth contribution of "Finance, insurance, real estate, rental, and leasing" was 0.2903 percentage points but it does not permit assertions about the values of the PGE and PCE components. This applies similarly to the other industries.

Finally, notice that the changes in PGE (0.3600 to 0.3565) and in PCE (−0.0696 to −0.0662) are small. However, the problem is in the fact that they change no matter the size. The changes in PGE and PCE with regrouping comes from the fact that Sum of PCE is not zero (0.3516 in row 17 or 0.3482 in row 18) without CIA. This non-zero Sum of PCE is a price effect "residual" that is unallocated to specific industries. It changes with regrouping and, hence, changes the values of PGE and PCE of the group members. These changes imply that the values of PGE and PCE are not unique and there is ambiguity in attributing them to specific industries.

Table 4 shows that Sum of PCE remains positive with regrouping in the year 2010. However, US GDP data during 1997-2015 show that Sum of PCE could be negative. Therefore, the null hypothesis that Sum of PCE = 0 is testable. Without regrouping, Sum of PCE ranged from −0.2727 to 0.3482 with a mean value of 0.1169 and standard deviation of 0.1429. With regrouping, the range was −0.2177 to 0.3523 with a mean value of 0.1279 and standard deviation of 0.1338. Sum of PCE = 0 can be rejected based on a t-value of 4.06, Pr(]T]>]t]) = 0.001, with regrouping and t-value of 3.47, Pr(]T]>]t]) = 0.003, without. Therefore, statistical evidence establishes that Sum of PCE is significantly different from zero in US GDP. Hence, PGE and PCE are not unique and there is ambiguity in attributing them to specific industries.

There is analytic basis for the above result that Sum of PCE is significantly different from zero in the US. To show this, recall the definition in (9) that Sum of PCE $\equiv X_t/X_{t-1} - \sum_j (Y_{t-1}^j/Y_{t-1})(X_t^j/X_{t-1}^j)$. That is, Sum of PCE $\neq 0$ in the US because $Y_{t-1}^j/Y_{t-1} \neq X_{t-1}^j/X_{t-1}$ and $\sum_j (Y_{t-1}^j/Y_{t-1}) = 1 \neq \sum_j (X_{t-1}^j/X_{t-1})$ because US GDP is not additive or $X_{t-1} \neq \sum_j X_{t-1}^j$. These two inequalities may account for why Sum of PCE appears large in proportion to US GDP growth. In Table 4, this proportion is on average around $(0.35/2.53)\times100=13.8\%$. That is, almost 14% of US GDP growth in 2010 is not allocated to industries.

The above analytic and statistical findings point to the conclusion that PGE and PCE in Table 4 for 2010 and also in other years are not unique for each industry in US GDP. This conclusion clouds the analysis of the role of industries in GDP growth, for example, in identifying industries subject to "supply-push" or "demand-pull" that basically depends on PCE in Tang and Wang's (2014) GDP growth analysis in Canada and in the US.

**Exactly Additive and Statistically Unique Quantity and Price Effects in Philippine GDP**

Philippine GDP in constant prices equals the simple sum of GDP of industries so that there are no residuals (Table 5). However, it yields Sum of PCE $\neq$ 0 (Table 6).

**Table 5. Gross Domestic Product and Employment in the Philippines, 2009-2010**

| | **GDP in current prices** (millions of pesos) | | **GDP in constant prices** (millions of 2000 pesos) | | **Employed persons** (thousands) | |
|---|---|---|---|---|---|---|
| | **2009** | **2010** | **2009** | **2010** | **2009** | **2010** |
| Agriculture, forestry, and fishery | 1,049,874 | 1,108,718 | 663,744 | 662,665 | 12043 | 11956 |
| Mining and quarrying | 106,396 | 128,727 | 59,130 | 65,898 | 166 | 199 |
| Manufacturing | 1,706,391 | 1,930,779 | 1,137,534 | 1,264,523 | 2,894 | 3,033 |
| Construction | 460,426 | 551,230 | 284,994 | 325,820 | 1,891 | 2,017 |
| Electricity, gas and water supply | 271,892 | 321,543 | 184,943 | 203,274 | 142 | 150 |
| Services (as one whole) | 4,431,165 | 4,962,483 | 2,966,895 | 3,179,358 | 17,925 | 18,682 |
| Transport Communication and Storage | 561,093 | 586,197 | 423,398 | 427,766 | 2,679 | 2,723 |
| Trade | 1,359,500 | 1,563,786 | 875,616 | 948,743 | 6,736 | 7,034 |
| Finance | 544,526 | 622,404 | 340,329 | 374,716 | 369 | 400 |
| Other Services | 1,966,045 | 2,190,096 | 1,327,552 | 1,428,133 | 8,141 | 8,525 |
| **Gross domestic product** | **8,026,143** | **9,003,480** | **5,297,240** | **5,701,539** | **35,060** | **36,037** |
| **Residual** | **0.0** | **0.0** | **0.0** | **0.0** | **0.0** | **0.0** |
| **Source:** Philippine Statistics Authority. | | | | | | |

| | Table 6. GDP Growth in the Philippines, 2009-2010 | | | |
|---|---|---|---|---|
| | | **PGE** | **PCE** | **PGE + PCE** |
| | GEAD growth contributions by industry | (Percentage points) | | |
| 1 | Agriculture, forestry, and fishery | -0.0213 | 0.1948 | 0.1735 |
| 2 | Mining and quarrying | 0.1517 | 0.0615 | 0.2133 |
| 3 | Manufacturing | 2.3734 | -0.5523 | 1.8211 |
| 4 | Construction | 0.8218 | 0.0313 | 0.8531 |
| 5 | Electricity, gas and water supply | 0.3358 | 0.1205 | 0.4563 |
| **6** | **Services (as one whole)** | **3.9536** | **0.1614** | **4.1150** |
| **7** | **Services (by category)** | **4.0281** | **0.0868** | **4.1150** |
| **8** | **Sum (1 to 5, and 6)** | **7.6150** | **0.0172** | **7.6323** |
| **9** | **Sum (1 to 5, and 7)** | **7.6896** | **-0.0573** | **7.6323** |
| **10** | **GDP Growth** | | | **7.6323** |

**Source**: Author's calculations of PGE for the *quantity* effect in (7) and PCE for the *price* effect in (8) from data in Table 5.

Consider row 6 in Table 6 which shows the contribution of "Services (as one whole)" when the four individual service industries in Table 5 are treated as one whole group. In comparison, row 7 shows the sum of the contributions of these four individual industries. The PGE contribution of 3.9536 percentage points of the group differs from the sum of the PGE contributions of 4.0281 percentage points of the group members. Similarly, the PCE contribution of 0.1614 percentage points of the group differs from the sum of the PCE contributions of 0.0868 percentage points of the group members. However, while they change with regrouping, the sum (PGE + PCE) remains the same, equal to 4.1150 percentage points (rows 6 and 7).

Rows 8 and 9 show that in 2010 Sum of PCE was −0.0573 when the service industries were treated individually and changed to 0.0172 when these services industries were treated as one whole. Quarterly GDP data during 2008-2015 showed that Sum of PCE ranged from −0.2445 to 0.1956 with a mean of −0.0192 and standard deviation of 0.1237 when the service industries were treated individually and ranged from −0.1704 to 0.2084 with a mean of 0.0023 and standard deviation of 0.1138 when the service industries were treated as one whole. Since the above ranges of Sum of PCE include zero, the results indicate that a statistical test of

Sum of PCE = 0 is in order. When the service industries were treated individually, the t-value was −0.62, Pr(]T]>]t]) = 0.543, and when the service industries were treated as one whole, the t-value was 0.08, Pr(]T]>]t]) = 0.938. These test results imply that Sum of PCE = 0 cannot be rejected, that is, Sum of PCE is not significantly different from zero in Philippine GDP.

There is analytic basis for the above empirical result that Sum of PCE is statistically zero in the Philippines. To show this, recall the definition in (9) that Sum of PCE ≡ $X_t/X_{t-1} - \sum_j \left(Y_{t-1}^j/Y_{t-1}\right)\left(X_t^j/X_{t-1}^j\right)$. If real GDP is in constant prices, as in the Philippines, this expression equals zero by replacing shares in nominal GDP, $Y_{t-1}^j/Y_{t-1}$, with shares of GDP in constant prices, $X_{t-1}^j/X_{t-1}$, and invoking additivity of GDP in constant prices or $X_t = \sum_j X_t^j$. Hence, $Y_{t-1}^j/Y_{t-1} \neq X_{t-1}^j/X_{t-1}$ but $\sum_j\left(Y_{t-1}^j/Y_{t-1}\right) = \sum_j\left(X_{t-1}^j/X_{t-1}\right) = 1$ so that Sum of PCE may be numerically different from zero in any year, although it appears very small as a proportion of Philippine GDP growth. Notice in Table 4 that this proportion is on average (in absolute value) around $(0.02/7.63)$×100=0.26%. That is, only ¼ of 1% of Philippine GDP growth in 2010 is not allocated to industries.

However, recall that statistical tests based on quarterly GDP data during 2008-2015 showed that the interpretation that Sum of PCE $= 0$ in the Philippines (Table 6) is warranted. Hence, the implications of the results in Italy apply to the Philippines. That is, PCE and PGE may be considered unique and attributable to specific industries in the Philippines.

Granted from above that PGE and PCE in Table 6 may be attributed to industries, it would be preferable to choose row 7 over row 6 for the growth contribution of "Services" because PGE and PCE in row 7 were obtained at a more disaggregated level compared to those in row 6. In this case, it may be asserted that the growth contribution of the above industry was

4.1150 percentage points of which 4.0281 came from PGE and 0.0868 came from PCE. Similar assertions are warranted for the other industries.

It may be recognized that the above statistical test findings may not generalize to countries with similar GDP in chained prices as in the US (e.g., Canada) or to countries with GDP in constant prices other than the Philippines. However, the findings underscore the importance of testing the hypothesis that Sum of PCE $= 0$—whenever Sum of PCE $\neq 0$ appears (Tables 4 & 6) —before interpreting PGE and PCE in industry contributions to GDP growth.

Finally, it is remarkable that PGE + PCE is the same between the two rows compared in above three tables, given by 0.3806 (Table 2, Italy), 0.2903 (Table 4, US), and 4.1150 (Table 6, Philippines). This means that in GEAD, the total contribution of an industry to GDP growth, PGE + PCE, is invariant with industry regrouping and this sum is attributable to a specific industry in all cases where PGE and PCE are unique (Italy); statistically unique (Philippines); and not unique (US). This invariance of the sum, PGE + PCE, of each industry insures exact additivity of the overall sum across industries to equal the economy's GDP growth.

**CIA Implications on ALP Growth Decomposition**

The preceding results from GDP growth decomposition by GEAD have implications on decomposition of ALP growth because ALP, by definition, is GDP ($X_t$) per unit of labor employment ($L_t$). To formalize the relation, let $Z_t$ be ALP and $Z_t^j$ be industry labor productivity. Recalling the GDP definition in (1), (2), and (3), $Z_t$ and $Z_t^j$ are defined by

$$Z_t \equiv \frac{X_t}{L_t} \quad ; \quad Z_t^j \equiv \frac{X_t^j}{L_t^j} \quad ; \quad L_t = \sum_j L_t^j \quad ; \quad l_t^j \equiv \frac{L_t^j}{L_t} \quad ; \quad \sum_j l_t^j = 1\,. \tag{20}$$

In (20), $l_t^j$ is an industry's share of total labor employment. Recall also that $X_t = \sum_j r_t^j X_t^j$ where $r_t^j \equiv P_{0,t}^j / P_{0,t}$. Therefore, (20) yields[6]

$$Z_t = \sum_j \frac{P_{0,t}^j}{P_{0,t}} \frac{L_t^j}{L_t} \frac{X_t^j}{L_t^j} = \sum_j r_t^j \, l_t^j \, Z_t^j \quad ; \quad Z_{t-1} = \sum_j r_{t-1}^j \, l_{t-1}^j \, Z_{t-1}^j \,. \tag{21}$$

It may be emphasized that (21) is *generally valid* because $X_t = Y_t / P_{0,t}$ and $X_t^j = Y_t^j / P_{0,t}^j$ are true by definition of real GDP as a deflated value whatever the formula for the deflators $P_{0,t}$ and $P_{0,t}^j$.

Let $G_t$ be the growth rate of $Z_t$ and $G_t^j$ be the growth rate of $Z_t^j$. That is,

$$G_t = \frac{Z_t - Z_{t-1}}{Z_{t-1}} \quad ; \quad G_t^j = \frac{Z_t^j - Z_{t-1}^j}{Z_{t-1}^j} \,. \tag{22}$$

Combining (20) to (22) yields Tang and Wang's (2004) GEAD formula for ALP growth[7]

$$G_t = \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} G_t^j + \sum_j \frac{Z_{t-1}^j}{Z_{t-1}} \left( r_t^j l_t^j - r_{t-1}^j l_{t-1}^j \right) G_t^j + \sum_j \frac{Z_{t-1}^j}{Z_{t-1}} \left( r_t^j l_t^j - r_{t-1}^j l_{t-1}^j \right). \tag{23}$$

Using the terminology in Dumagan (2013a), an industry's contribution to ALP growth consists of

$$\text{WSPGE (within-sector productivity growth effect)} \equiv \frac{Y_{t-1}^j}{Y_{t-1}} G_t^j \,; \tag{24}$$

$$\text{DSRE (dynamic structural reallocation effect)} \equiv \frac{Z_{t-1}^j}{Z_{t-1}} \left( r_t^j l_t^j - r_{t-1}^j l_{t-1}^j \right) G_t^j \,; \tag{25}$$

$$\text{SSRE (static structural reallocation effect)} \equiv \frac{Z_{t-1}^j}{Z_{t-1}} \left( r_t^j l_t^j - r_{t-1}^j l_{t-1}^j \right). \tag{26}$$

[6] For an analysis of the role of relative prices in alternative labor productivity growth decompositions, see Dumagan (2013b), Diewert (2015), and Dumagan & Balk (2016).

[7] As noted earlier in the text, Dumagan (2013a) first used the acronym GEAD – "generalized exactly additive decomposition" – to describe (23) because it yields "actual" ALP growth regardless of the index formulas underlying GDP, that is, given GDP either in chained or in constant prices.

WSPGE is the industry's labor productivity growth weighted by its share in nominal GDP. DSRE and SSRE are reallocation effects due to combined effects of changes in relative prices and labor shares.[8] DSRE is related to the *Baumol effect* that indicates Baumol's (1967) "growth disease" when resources are absorbed by "stagnant" industries, that is, those with low values of $(Z_{t-1}^j/Z_{t-1})G_t^j$, since they could have high values of $(r_t^j l_t^j - r_{t-1}^j l_{t-1}^j)$ from increasing labor shares, given relative prices. SSRE is related to the Denison (1962) effect which shows a labor reallocation effect on ALP growth from changes in industry labor shares, given relative prices.

As a first step to examine uniqueness, add DSRE in (25) to SSRE in (26) to obtain

$$\text{ISRE (inter-sector reallocation effect)} \equiv \text{DSRE} + \text{SSRE} = \frac{Z_t^j}{Z_{t-1}}\left(r_t^j l_t^j - r_{t-1}^j l_{t-1}^j\right). \quad (27)$$

Therefore, (23) to (27) may be combined to yield

$$G_t = \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}} G_t^j + \sum\nolimits_j \frac{Z_t^j}{Z_{t-1}}\left(r_t^j l_t^j - r_{t-1}^j l_{t-1}^j\right) = \text{Sum of WSPGE} + \text{Sum of ISRE}. \quad (28)$$

By using (20) and (21), (28) becomes

$$G_t = \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}} G_t^j + \left(\frac{X_t/X_{t-1}}{L_t/L_{t-1}} - \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}} \frac{X_t^j/X_{t-1}^j}{L_t^j/L_{t-1}^j}\right). \quad (29)$$

To evaluate the expression in parentheses, note that no labor reallocation means that labor shares remain the same, that is, $L_{t-1}^j/L_{t-1} = L_t^j/L_t$ so that $L_t/L_{t-1} = L_t^j/L_{t-1}^j$. In this case, (29) yields

$$G_t = \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}} G_t^j + \frac{L_{t-1}}{L_t}\left(\frac{X_t}{X_{t-1}} - \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}} \frac{X_t^j}{X_{t-1}^j}\right). \quad (30)$$

It is interesting to note that the term inside parentheses in (30) is exactly the expression for Sum of PCE in (9). Therefore, without labor reallocation, (29) and (30) become

[8] Dumagan (2013a) adopted the terms WSPGE, DSRE, and SSRE from the ALP growth decomposition by the Asian Development Bank (2010) where, in contrast, relative prices are absent. In the terminology by Nordhaus (2002), WSPGE corresponds to *pure productivity growth effect*; DSRE to the Baumol (1967) effect and SSRE to the Denison (1962) effect noted above.

$$G_t = \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} G_t^j + \frac{L_{t-1}}{L_t}(\text{Sum of PCE}) \quad ; \quad \text{Sum of ISRE} = \frac{L_{t-1}}{L_t}(\text{Sum of PCE}) . \quad (31)$$

It follows from (31) that when (28) is applied to ALP growth decomposition in Italy where GDP is in chained prices and the GDP index is the Laspeyres quantity index—which yields Sum of PCE = 0—then Sum of ISRE = 0 if there is no labor reallocation. Therefore, the result that Sum of ISRE ≠ 0 in Table 7 must be due only to labor reallocation effects.

**Table 7. ALP Growth in Italy, 2009-2010**

| | | WSPGE | ISRE | WSPGE + ISRE |
|---|---|---|---|---|
| | GEAD growth contributions by industry | (Percentage points) | | |
| 1 | Agriculture, hunting and forestry; fishing and operation of fish hatcheries and fish farms | -0.0120 | 0.0630 | 0.0510 |
| 2 | Industry, including energy | 1.6836 | -0.9253 | 0.7583 |
| 3 | Construction | -0.1275 | 0.0260 | -0.1015 |
| 4 | Wholesale and retail trade, repair of motor vehicles and household goods, hotels and restaurants; transport and communication | 0.7253 | -0.3121 | 0.4132 |
| 5 | Financial, real estate, renting and business activities | 0.0076 | 0.5535 | 0.5611 |
| **6** | **Other service activities (as one whole)** | **-0.0939** | **0.6403** | **0.5464** |
| **7** | **Other service activities (by category)** | **-0.0018** | **0.5482** | **0.5464** |
| **8** | **Sum (1 to 5, and 6)** | **2.1831** | **0.0453** | **2.2284** |
| **9** | **Sum (1 to 5, and 7)** | **2.2752** | **-0.0468** | **2.2284** |
| **10** | **ALP growth** | | | **2.2284** |

**Source**: Author's calculations of WSPGE in (24) and ISRE in (27) from data in Table 1.

The above analysis means that while ISRE at the industry level is due to the combined effects of a change in labor share and a change in relative price, Sum of ISRE includes only the effects of changes in labor shares, that is, labor reallocation effects. Since Sum of PCE = 0, there are no unallocated price effects to make ISRE ambiguous. Hence, by implication, WSPGE and ISRE in Table 7 are attributable to industries in the same way that PGE and PCE in Table 2 are attributable to the same industries. This follows because PGE corresponds to WSPGE and PCE corresponds to ISRE. In fact, these two tables will have identical results if labor is removed from the calculations in Table 7.

It follows from above that, for example, it can be asserted that "Other service activities" contributed 0.5464 percentage points to the 2.2284% ALP growth in Italy in 2010. Notice, however, that WSPGE and ISRE change between rows 6 and 7 in Table 7 although their sum remains 0.5464. In contrast, there is no corresponding change in PGE and PCE between rows 6 and 7 in Table 2. To explain this contrast, note that the aggregation of GDP does not affect GDP growth contributions in rows 6 and 7 of Table 2 because the underlying Laspeyres quantity index is CIA. However, row 6 of Table 7 involves GDP aggregation and labor aggregation to obtain the contributions of "other service activities (as one whole)." In this case, the "correct" WSPGE and ISRE are those in row 7 based on the principle that growth contributions should be calculated at the more disaggregated level compared to row 6. Therefore, in this example, it can be asserted that WSPGE is $-0.0018$ and ISRE is 0.5482 for a total contribution by "other service activities" of 0.5464 percentage points to ALP growth in Italy.

In contrast, in the US, Sum of ISRE $\neq 0$ in Table 8 can be interpreted as not due only to labor reallocation effects because there is strong statistical evidence that Sum of PCE $\neq 0$ in US GDP, implying that PCE changes with industry regrouping. Since PCE corresponds to ISRE, it follows that ISRE changes accordingly because PCE changes even if there is no labor reallocation. That is, for each industry, ISRE is tainted by the fact that the Fisher index is not CIA, making ISRE as ambiguous as PCE. By implication, WSPGE is also ambiguous because WSPGE +ISRE is given for each industry. Hence, in Table 8, it can be asserted that "Finance, insurance, real estate, rental, and leasing," for example, contributed 0.4439 percentage points (row 15 or row 16) to ALP growth of 3.3177% in the US but it is not clear or ambiguous if the "correct" WSPGE and ISRE components are those in row 15 or in row 16. Therefore, in general, the sum WSPGE + ISRE in the last column of Table 8 can be asserted as "correct" for each industry but no such assertion can be made for the individual WSPGE and ISRE.

**Table 8. ALP Growth in the United States, 2009-2010**

| | | WSPGE | ISRE | WSPGE + ISRE |
|---|---|---|---|---|
| | GEAD growth contributions by industry | (Percentage points) | | |
| 1 | Agriculture, forestry, fishing, and hunting | 0.0040 | 0.1470 | 0.1510 |
| 2 | Mining | -0.1621 | 0.4387 | 0.2766 |
| 3 | Utilities | 0.1985 | -0.0946 | 0.1039 |
| 4 | Construction | 0.0708 | -0.3356 | -0.2648 |
| 5 | Manufacturing | 0.9745 | -0.3119 | 0.6626 |
| 6 | Wholesale trade | 0.2897 | -0.0002 | 0.2894 |
| 7 | Retail trade | 0.2023 | -0.0446 | 0.1576 |
| 8 | Transportation and warehousing | 0.2079 | -0.0390 | 0.1689 |
| 9 | Information | 0.4311 | -0.2815 | 0.1495 |
| 10 | Professional and business services | 0.2298 | 0.1911 | 0.4209 |
| 11 | Educational services, health care, and social assistance | -0.0849 | 0.2846 | 0.1997 |
| 12 | Arts, entertainment, recreation, accommodation, and food services | 0.1422 | -0.0317 | 0.1105 |
| 13 | Other services, except government | 0.0069 | 0.0027 | 0.0096 |
| 14 | Government | 0.1430 | 0.2893 | 0.4323 |
| **15** | **Finance, insurance, real estate, rental, and leasing (as one whole)** | **0.6380** | **-0.1940** | **0.4439** |
| **16** | **Finance, insurance, real estate, rental, and leasing (by category)** | **0.5772** | **-0.1333** | **0.4439** |
| **17** | **Sum (1 to 14, and 15)** | **3.2916** | **0.0201** | **3.3117** |
| **18** | **Sum (1 to 14, and 16)** | **3.2309** | **0.0809** | **3.3117** |
| **19** | **ALP growth** | | | **3.3117** |
| | **Source**: Author's calculations of WSPGE in (24) and ISRE in (27) applied to data in Table 3. | | | |

However, in the Philippines, Sum of ISRE ≠ 0 in Table 9 can be treated as due only to labor reallocation effects because Sum of PCE = 0 is statistically warranted in Philippine GDP. This statistical finding implies that WSPGE and ISRE are attributable to individual industries.

**Table 9. ALP Growth in the Philippines, 2009-2010**

| | | WSPGE | ISRE | WSPGE + ISRE |
|---|---|---|---|---|
| | GEAD growth contributions by industry | (Percentage points) | | |
| 1 | Agriculture, forestry, and fishery | 0.0735 | -0.2592 | -0.1857 |
| 2 | Mining and quarrying | -0.0932 | 0.2648 | 0.1715 |
| 3 | Manufacturing | 1.2864 | -0.0909 | 1.1955 |
| 4 | Construction | 0.4121 | 0.2624 | 0.6745 |
| 5 | Electricity, gas and water supply | 0.1434 | 0.2087 | 0.3521 |
| **6** | **Services (as one whole)** | **1.5555** | **0.9515** | **2.5070** |
| **7** | **Services (by category)** | **1.3738** | **1.1332** | **2.5070** |
| **8** | **Sum (1 to 5, and 6)** | **3.3776** | **1.3373** | **4.7150** |
| **9** | **Sum (1 to 5, and 7)** | **3.1959** | **1.5191** | **4.7150** |
| **10** | **ALP Growth** | | | **4.7150** |
| | **Source**: Author's calculations of WSPGE in (24) and ISRE in (27) applied to data in Table 5. | | | |

Granted from above that WSPGE and ISRE in Table 9 may be attributed to industries, it would be preferable to choose row 7 over row 6 for the growth contribution of "Services" because WSPGE and ISRE in row 7 were obtained at a more disaggregated level compared to those in row 6. In this case, it may be asserted that the contribution to ALP growth of the above industry was 2.5070 percentage points of which 1.3738 came from WSPGE and 1.1332 came from ISRE. Similar assertions are warranted for the other industries.

At this juncture, it may be remarked that the issue of uniqueness of PGE and PCE pertains only to GDP growth decomposition (Tables 2, 4, and 6). This issue does not pertain to ALP growth decomposition (Tables 7, 8, and 9) because WSPGE is the effect of a change in labor share combined with PGE while ISRE is the effect of a change in labor share combined with PCE.

Finally, it is remarkable that the sum WSPGE + ISRE remains the same between the two rows compared in above three tables, given by 0.5464 (Table 7, Italy), 0.4439 (Table 8, US), and 2.5070 (Table 9, Philippines). This means that in GEAD the total contribution of an industry to ALP growth, given by WSPGE + ISRE, is invariant with industry regrouping and this total is attributable to specific industries. This invariance of the above sum insures exact additivity of growth contributions across industries to equal ALP growth.

## Conclusion

In the GEAD framework for GDP growth, an industry's contribution consists of a quantity effect from real GDP growth, PGE, and a price effect from a change in relative price, PCE. This paper affirms that the total contribution of an industry is PGE + PCE and that their overall sum across industries exactly equals actual growth of GDP either in chained or in constant prices. That is, “exact additivity” of PGE and PCE and their “generality” to any real GDP are not in question.

However, this paper points out that “uniqueness” of PGE and PCE is an analytic issue with important practical implications that has not been examined in GEAD. Uniqueness is important because it permits unambiguous attributions of values of the above effects to specific industries, which would not be possible otherwise. In cases without uniqueness, there is ambiguity in the magnitude of PGE and PCE that translates to incorrect inferences and possibly leads to misguided growth policies.

The results of this paper showed that individual PGE and PCE components are analytically and empirically “unique” and, hence, unquestionably attributable to specific industries only if GDP is in chained prices and the GDP index is CIA like the Laspeyres quantity index in the case of Italian GDP. In contrast, analytic and empirical results show that PGE and PCE are not unique and their attributability to industries are questionable when (1) GDP is in chained prices and the GDP index is not CIA like the Fisher quantity index in the case of the US, or (2) GDP is in e constant prices based on the direct Laspeyres quantity index, as in the Philippines. However, the analytic and empirical results show that the problem is more severe in case (1) or milder in case (2). To resolve the above issue for practical purposes, this paper showed by statistical tests that PGE and PCE are not unique and, therefore, not unambiguously attributable to industries in case (1) while they can be treated as unique and attributable to industries in case (2).

The above statistical findings may not be generalizable to countries with GDP in chained prices like that in the US (e.g., Canada) or to countries other than the Philippines with GDP in constant prices. However, they underscore the importance of testing the hypothesis that

Sum of PCE = 0—whenever Sum of PCE ≠ 0 appears from GDP data—prior to analyzing PGE and PCE in industry contributions to GDP growth.

By definition, ALP growth comes from GDP growth and changes in labor shares of industries. Hence, in the GEAD framework, an industry's contribution to ALP growth consists of WSPGE and ISRE, where WSPGE is the effect of a change in labor share combined with PGE from GDP growth, and ISRE is the effect of a change in labor share combined with PCE from change in relative price. This paper affirms that in GEAD the total contribution of an industry is WSPGE + ISRE to growth of ALP when GDP is in chained or in constant prices. However, the attribution of WSPGE and ISRE to industries is subject to the same qualifications as the attribution of PGE and PCE because the attributes of PGE affect WSPGE, on the one hand, and the attributes of PCE affect ISRE, on the other. Therefore, WSPGE and ISRE are attributable without question to specific industries in countries like Italy where GDP is in chained prices based on the Laspeyres quantity index. Hence, the statistical tests showing that PGE and PCE are questionable for attribution to industries when GDP is in chained prices based on the Fisher index, for example, in the US, imply that WSPGE and ISRE are also questionable in the same vein. By similar reasoning, the statistical tests showing that PGE and PCE can be considered attributable to industries when GDP is in constant prices, for example, in the Philippines, imply that WSPGE and ISRE are also attributable to the same industries.

Finally, this paper chose Italian, US, and Philippine GDP data to illustrate empirically the analytic results because these three sets of GDP represent current official practices in compiling real GDP in all countries. Therefore, the results of this paper have global practical relevance.

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