Effects of Change in Relative Prices in Existing Decompositions of Aggregate Labor Productivity Growth: A Resolution of the Aggregate Effect

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Abstract

Diewert (2015) reworked Tang and Wang's (2004) growth decomposition and claimed that: "Thus even if all industry labor productivity levels remain constant and all labor input shares remain constant, economy wide labor productivity growth *can change due to changes in industry real output prices* (italics added)" (p. 370). However, contrary to his 2015 claim, Diewert (2016) found "puzzling" results from Australian data where the sum of price change effects across industries did not matter much and explained this puzzle by an *approximation* formula that showed price effects sum to zero with the first-order accuracy. In contrast, this paper derives the *exact* formula that shows price effects sum to zero, depending on the quantity index underlying the GDP in the definition of aggregate labor productivity. It is shown that Diewert's formula is an approximation to this paper's exact formula showing that the aggregate effect of relative price changes is zero.

Keywords: Labor productivity growth, GDP growth, relative prices, index number theory

JEL classification: C43, O47

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Tang and Wang (2004) introduced *relative price*, the ratio of an industry's GDP deflator to the economy's GDP deflator as the real price of each industry's GDP—making the economy's "GDP basket" as numeraire—to obtain "exactly additive" contributions to aggregate labor productivity (ALP) growth given GDP in constant or in chained prices (i.e., "general"). Thus, Dumagan (2013) called Tang and Wang's ALP growth decomposition a generalized exactly additive decomposition (GEAD).

Diewert (2010, 2015) reworked Tang and Wang's decomposition to highlight the effects on ALP growth of changes in industry *labor productivities*, *labor shares*, and *relative prices* individually as well as jointly. Thus, if all industry labor productivities and all labor shares remain constant while industry relative prices change, Diewert claimed that ALP growth could change. In this case, this paper shows that Diewert's claim can happen if the weight of an industry's contribution to ALP growth is inappropriate. It is shown that the appropriate weight is the weight of the industry's contribution to the growth of the specific quantity index underlying GDP. With the appropriate weight, the positive, zero, or negative effects of relative price changes on industry growth contributions will sum to zero and leave ALP growth unchanged even when there are changes in industry labor productivities and labor shares.

The second section of this paper reproduces Diewert's (2015) alternative GEAD decomposition of ALP growth, taking off from the real GDP aggregation using relative price weights in Tang and Wang's (2004) original GEAD decomposition. However, as these decompositions are essentially the same, they will henceforth be referred to in this paper as the Tang-Wang-Diewert (TWD, in chronological order) decomposition or simply TWD for brevity.

The third section shows the effects of relative price changes in TWD in two cases: the *special* case where industry labor productivities and labor shares are all constant and the *general* case where they change. It is shown in each case that if the *weight* of an industry is the weight of its contribution to the growth of the quantity index underlying aggregate GDP, the effects of price changes will cancel out and leave ALP growth unchanged. This weight is appropriate because it makes the aggregate GDP index equal the weighted sum of quantity relatives where the sum of weights equals one, which is the way the above index is constructed in the first place.

Empirical results from country applications illustrate the effects on ALP growth and GDP growth of relative price changes from using appropriate weights when the quantity index underlying GDP is *chained Fisher* (United States), *direct Laspeyres* (Philippines), and *chained Laspeyres* (Italy). These results have worldwide implications because the above quantity indexes underlie GDP in current practice in all countries. Moreover, the implications of this paper's findings on earlier applications of TWD where weights are "inappropriate" in some cases are indicated. Examples of these applications are Tang and Wang (2004, 2014) to Canada and United States; Diewert (2015, 2016) to Australia; Dumagan (2013) to the Philippines, Thailand, and United States; and Dumagan (2017) to Italy, Philippines, and United States.

The fourth section concludes this paper.

The Tang-Wang-Diewert (TWD) Decomposition of ALP Growth

Using the notation in Diewert (2015), let Y represent real GDP. Also, let P be the *implicit deflator*, the ratio of nominal GDP to Y. The superscript t denotes time, $t = (0,1)$, and the subscript $n = 1, 2, \cdots, N$ represents an industry. As nominal GDP is the product of real GDP and its corresponding deflator and nominal GDP is additive, it follows that

$$P^t Y^t = \sum_{n=1}^{N} P_n^t Y_n^t \quad ; \quad p_n^t \equiv \frac{P_n^t}{P^t} \quad ; \quad Y^t = \sum_{n=1}^{N} p_n^t Y_n^t . \qquad (1)$$

In (1), p_n^t is a relative price, the ratio of an industry's GDP deflator to the economy's GDP deflator. As such, p_n^t is a real price that converts Y_n^t to $p_n^t Y_n^t$ in the *same* unit across industries as Y^t (i.e., "GDP basket" is the numeraire). Therefore, $p_n^t Y_n^t$ is additive regardless of the original units of Y_n^t.

Also, let L represents labor and X represents labor productivity, the ratio of Y to L. That is,

$$X^t \equiv \frac{Y^t}{L^t} \quad ; \quad X_n^t \equiv \frac{Y_n^t}{L_n^t} \quad ; \quad L^t = \sum_{n=1}^{N} L_n^t \quad ; \quad s_{L_n}^t \equiv \frac{L_n^t}{L^t} \quad ; \quad \sum_{n=1}^{N} s_{L_n}^t = 1\,. \tag{2}$$

The values of Y_n^t, Y^t, and their corresponding nominal values together with labor employment levels, L_n^t, L^t, are publicly available from national accounts. The relative prices, $p_n^t \equiv P_n^t / P^t$, come from the implicit deflators, P_n^t, P^t, which are the ratios of nominal GDP to real GDP.

Finally, (1) and (2) yield ALP, X^t, which is

$$X^t = \sum_{n=1}^{N} \frac{P_n^t}{P^t} \frac{L_n^t}{L^t} \frac{Y_n^t}{L_n^t} = \sum_{n=1}^{N} p_n^t s_{L_n}^t X_n^t. \tag{3}$$

Diewert (2015) took off from Tang and Wang's (2004) equations (1) to (3).

Because Diewert's claim motivated this paper, it is convenient to start with his framework below. Let t change from 0 to 1. Hence, (1) to (3) yield

$$\frac{X^1}{X^0} = \frac{\sum_{n=1}^{N} p_n^1 s_{L_n}^1 X_n^1}{\sum_{n=1}^{N} p_n^0 s_{L_n}^0 X_n^0} = \frac{\sum_{n=1}^{N} \left(\frac{p_n^1}{p_n^0} \frac{s_{L_n}^1}{s_{L_n}^0} \frac{X_n^1}{X_n^0} \right) \left(p_n^0 s_{L_n}^0 X_n^0 \right)}{\sum_{n=1}^{N} p_n^0 s_{L_n}^0 X_n^0} \quad ; \quad p_n^0 s_{L_n}^0 X_n^0 = \frac{P_n^0}{P^0} \frac{Y_n^0}{L^0}\,; \tag{4}$$

$$s_{Y_n}^0 \equiv \frac{p_n^0 s_{L_n}^0 X_n^0}{\sum_{n=1}^{N} p_n^0 s_{L_n}^0 X_n^0} = \frac{P_n^0 Y_n^0}{\sum_{n=1}^{N} P_n^0 Y_n^0} \quad ; \quad \sum_{n=1}^{N} s_{Y_n}^0 = 1 \quad ; \quad \frac{X^1}{X^0} = \sum_{n=1}^{N} s_{Y_n}^0 \left(\frac{p_n^1}{p_n^0} \frac{s_{L_n}^1}{s_{L_n}^0} \frac{X_n^1}{X_n^0} \right); \tag{5}$$

$$\text{ALP growth} \equiv \frac{X^1}{X^0} - 1 = \sum_{n=1}^{N} s_{Y_n}^0 \left[\left(\frac{p_n^1}{p_n^0} \frac{s_{L_n}^1}{s_{L_n}^0} \frac{X_n^1}{X_n^0} \right) - 1 \right]. \tag{6}$$

The growth rates of relative price, labor share, and labor productivity are defined by

$$\rho_n \equiv \frac{p_n^1}{p_n^0} - 1 \quad ; \quad \sigma_n \equiv \frac{s_{L_n}^1}{s_{L_n}^0} - 1 \quad ; \quad \gamma_n \equiv \frac{X_n^1}{X_n^0} - 1 \,. \tag{7}$$

Combining (6) and (7) yields

$$\text{ALP growth} \equiv \frac{X^1}{X^0} - 1 = \sum_{n=1}^{N} s_{Y_n}^0 \left[(1+\rho_n)(1+\sigma_n)(1+\gamma_n) - 1\right] \tag{8}$$

$$= \sum_{n=1}^{N} s_{Y_n}^0 \left(\rho_n + \sigma_n + \gamma_n + \rho_n\sigma_n + \rho_n\gamma_n + \sigma_n\gamma_n + \rho_n\,\sigma_n\gamma_n\right). \tag{9}$$

Diewert's decomposition in (9) may be shown to yield Tang and Wang's decomposition. For this purpose, regroup (9) as

$$\frac{X^1}{X^0} - 1 = \sum_{n=1}^{N} s_{Y_n}^0 \gamma_n + \sum_{n=1}^{N} s_{Y_n}^0 \left(\rho_n + \sigma_n + \rho_n\sigma_n + \rho_n\gamma_n + \sigma_n\gamma_n + \rho_n\,\sigma_n\gamma_n\right). \tag{10}$$

In Dumagan's (2018a) terminology, the first sum in (10) is Sum of WSPGE where WSPGE is *with-in sector productivity growth effect* and the second is Sum of ISRE where ISRE is *inter-sector reallocation effect.* Hence, substituting the definitions in (7) into (10) yields

$$\text{Sum of WSPGE} \equiv \sum_{n=1}^{N} s_{Y_n}^0 \gamma_n = \sum_{n=1}^{N} s_{Y_n}^0 \left(\frac{X_n^1}{X_n^0} - 1\right) = \sum_{n=1}^{N} s_{Y_n}^0 \left(\frac{Y_n^1/L_n^1}{Y_n^0/L_n^0} - 1\right). \tag{11}$$

Moreover, it can be verified that

$$\text{Sum of ISRE} \equiv \sum_{n=1}^{N} s_{Y_n}^0 \left(\rho_n + \sigma_n + \rho_n\sigma_n + \rho_n\gamma_n + \sigma_n\gamma_n + \rho_n\,\sigma_n\gamma_n\right) \tag{12}$$

$$= \sum_{n=1}^{N} s_{Y_n}^0 \left(\frac{p_n^1}{p_n^0}\frac{s_{L_n}^1}{s_{L_n}^0} - 1\right)\frac{X_n^1}{X_n^0} = \sum_{n=1}^{N} \left(\frac{p_n^1 Y_n^1/L^1}{Y^0/L^0} - s_{Y_n}^0 \frac{Y_n^1/L_n^1}{Y_n^0/L_n^0}\right). \tag{13}$$

It follows from (10) to (13) that Diewert's decomposition in (9) yields

$$\frac{X^1}{X^0} - 1 = \sum_{n=1}^{N} s_{Y_n}^0 \left(\frac{Y_n^1/L_n^1}{Y_n^0/L_n^0} - 1 \right) + \sum_{n=1}^{N} \left(\frac{p_n^1 Y_n^1/L^1}{Y^0/L^0} - s_{Y_n}^0 \frac{Y_n^1/L_n^1}{Y_n^0/L_n^0} \right). \qquad (14)$$

It is important to note that (14) is equivalent to Tang and Wang's (2004) original decomposition.[1] Because Diewert's (9) equals Tang and Wang's (14), the above framework will henceforth be called the Tang-Wang-Diewert ALP growth decomposition that for brevity will be referred to as TWD for expository purposes in this paper. To avoid confusion, TWD is exactly the same as GEAD, which has been used earlier (Dumagan, 2013, 2017, 2018a) to refer to Tang and Wang's decomposition.

In the following analysis, $s_{Y_n}^0$ plays a central role. As shown in (9), $s_{Y_n}^0$ is the weight of an industry's contribution to ALP growth in TWD and by definition in (5), $s_{Y_n}^0$ is an industry's share of GDP in current prices. However, this paper argues below that $s_{Y_n}^0$ as defined above is not necessarily the appropriate weight in all cases. The appropriate one should be specific to the quantity index formula in current practice—namely, chained Laspeyres, direct Laspeyres, or chained Fisher—underlying the type of real GDP in the ALP growth decomposition.[2]

Relative Price Change Effects on ALP Growth

It will be shown that if the weights are appropriate, the effects of relative price changes will cancel out and leave ALP growth unchanged in Diewert's special case where industry labor productivities and labor shares are all constant and in the general case where they all change.

Relative Price Change Effects in Diewert's Special Case

In Diewert's special case, let an industry's contribution to ALP growth from a change in its relative price be $c_n^{p(0,1)}$ and let the sum of all contributions be $C^{p(0,1)}$. Hence, (7) and (9) yield

[1] Equation (14) above is equivalent to Tang and Wang's (2004, p. 426) Equation (3).

[2] See Balk (2010) for a discussion of direct and chained indexes.

$$c_n^{p(0,1)} \equiv s_{Y_n}^0 \left(\frac{p_n^1}{p_n^0} - 1\right) \quad ; \quad C^{p(0,1)} \equiv \sum_{n=1}^{N} c_n^{p(0,1)} = \sum_{n=1}^{N} s_{Y_n}^0 \left(\frac{p_n^1}{p_n^0} - 1\right). \tag{15}$$

In this special case, industry labor productivities and labor shares are all constant, thus (2) implies

$$\frac{Y_n^0}{L_n^0} = \frac{Y_n^1}{L_n^1} \Longrightarrow \frac{L_n^1}{L_n^0} = \frac{Y_n^1}{Y_n^0} \quad ; \quad \frac{L_n^0}{L^0} = \frac{L_n^1}{L^1} \Longrightarrow \frac{L^1}{L^0} = \frac{L_n^1}{L_n^0} \quad ; \quad \frac{Y_n^1}{Y_n^0} = \frac{L^1}{L^0}. \tag{16}$$

In turn, the last equality in (16) implies $(Y_n^1/Y_n^0)(L^0/L^1) = 1$. This result together with the GDP aggregation in (1) and the share definition in (5) transforms (15) to

$$C^{p(0,1)} \equiv \sum_{n=1}^{N} s_{Y_n}^0 \left(\frac{p_n^1}{p_n^0} - 1\right) \frac{Y_n^1}{Y_n^0} \frac{L^0}{L^1} = \frac{L^0}{L^1} \sum_{n=1}^{N} \left(\frac{p_n^1 Y_n^1}{Y^0} - s_{Y_n}^0 \frac{Y_n^1}{Y_n^0}\right)$$

$$= \frac{L^0}{L^1} \left(\frac{Y^1}{Y^0} - \sum_{n=1}^{N} s_{Y_n}^0 \frac{Y_n^1}{Y_n^0}\right). \tag{17}$$

In this special case of constant labor productivities, (11) and (16) yield Sum of WSPGE $= 0$ and Sum of ISRE in (13) equals $C^{p(0,1)}$ in (17). That is,

$$\text{ISRE} = c_n^{p(0,1)} \equiv \frac{L^0}{L^1} \left(\frac{p_n^1 Y_n^1}{Y^0} - s_{Y_n}^0 \frac{Y_n^1}{Y_n^0}\right); \tag{18}$$

$$\text{Sum of ISRE} = C^{p(0,1)} \equiv \sum_{n=1}^{N} c_n^{p(0,1)} = \frac{L^0}{L^1} \left(\frac{Y^1}{Y^0} - \sum_{n=1}^{N} s_{Y_n}^0 \frac{Y_n^1}{Y_n^0}\right). \tag{19}$$

As may be seen in (15), $c_n^{p(0,1)}$ could be positive, zero, or negative for an individual industry. In this case, Diewert's claim is equivalent to saying that $C^{p(0,1)} \neq 0$ is possible if $p_n^1/p_n^0 \neq 1$ for all or some n. This claim appears "obvious" from $C^{p(0,1)}$ in (15), but it turns out that there is an appropriate $s_{Y_n}^0$ for each of the three types of real GDP in current practice worldwide and each one makes the equivalent $C^{p(0,1)}$ in (19) equal to zero.

If the quantity index underlying GDP is consistent-in-aggregation (CIA), then (19) yields

$$C^{p(0,1)} \equiv \frac{L^0}{L^1}\left(\frac{Y^1}{Y^0} - \sum_{n=1}^{N} s_{Y_n}^0 \frac{Y_n^1}{Y_n^0}\right) = 0 \quad ; \quad \frac{Y^1}{Y^0} = \sum_{n=1}^{N} s_{Y_n}^0 \frac{Y_n^1}{Y_n^0} \quad \text{if } \frac{Y^1}{Y^0} \text{ is CIA} . \qquad (20)$$

To explain the above result, note that Y^1/Y^0 is the implicit aggregate GDP quantity index whereas Y_n^1/Y_n^0 is the implicit GDP quantity index of an industry. If the aggregate index is CIA, the expression inside the parentheses equals zero, implying that $C^{p(0,1)}$ is zero. This follows from the definition of CIA that the aggregate index equals the weighted sum of the industry indexes and the weights sum to one (Balk, 1996; Diewert, 1978; Vartia, 1976). In this case, relative price changes affect only the contributions of each industry but the effects sum to zero and, thus, do not contribute to ALP growth, contrary to Diewert's claim.

The aggregate index underlying Y^1/Y^0 is defined such that it equals the weighted sum of the quantity relatives (i.e., relative changes in quantities) where the sum of weights equals one. This definition applies to indexes with or without the CIA property. As noted below, it is satisfied by the "additive decomposition" of the Fisher index, which is not CIA, using the most detailed quantity relatives. However, under CIA, the definition is satisfied even when quantity relatives are replaced by sub-aggregate GDP indexes, for example, by industry GDP indexes like Y_n^1/Y_n^0.

The appropriate $s_{Y_n}^0$ is the one that satisfies the above definition and, by implication, is also the one that yields $C^{p(0,1)} = 0$. In this light, Dumagan (2018a) showed that the appropriate $s_{Y_n}^0$ is the *share of GDP in current prices* defined in (5) if the GDP quantity index is chained Laspeyres, but it is the share of GDP in constant prices if the above index is direct Laspeyres. That is, $s_{Y_n}^0$ should be specific to the GDP in the numerator of ALP.

The advantage of CIA is that data on $Y_n^0, Y^0, Y_n^1,$ and Y^1 from published national accounts yield the $s_{Y_n}^0$ that satisfy (20). This is true for GDP based on chained Laspeyres in the European

Union (2007) and for GDP in constant prices based on direct Laspeyres in other countries. Conversely, the disadvantage of lack of CIA is that Y_n^0, Y^0, Y_n^1, and Y^1 from GDP based on chained Fisher in Canada and the United States (Chevalier, 2003; Fisher, 1922; Landefeld & Parker, 1997) also yield $s_{Y_n}^0$ but this $s_{Y_n}^0$ does not satisfy (20) and, therefore, is inappropriate.

However, CIA is not necessary for (20) to be zero.[3] In the case of Canadian and U.S. GDPs that are based on the chained Fisher index, which is not CIA, Dumagan (2018a) also showed that the expression inside the square brackets in (20) equals zero by using the additive decomposition of the Fisher index (Balk, 2004; Dumagan, 2002; Van IJzeren, 1952) that the U.S. Bureau of Economic Analysis (BEA) uses to determine contributions to U.S. GDP growth (Moulton & Seskin, 1999). In this additive decomposition, the "Fisher weight" should be used in place of $s_{Y_n}^0$. However, to implement this Fisher additive decomposition given that the Fisher index is not CIA, the data needed to satisfy (20) are the most detailed components of Y_n^0, Y_n^1 down to the commodity-level prices and quantities used to compute the Fisher index in the first place. Unfortunately, these detailed price and quantity data are not publicly available from the national accounts published by the U.S. BEA and Statistics Canada. Therefore, the Fisher index also satisfies (20) but the necessary data are not publicly available. Thus, in principle, (20) equals zero in all countries because the chained Laspeyres, direct Laspeyres, and chained Fisher are the indexes used in current GDP practice worldwide.

An industry's price change contribution given by $c_n^{p(0,1)}$ in (19) can be computed by decomposing GDP growth, which is a special case of ALP growth when each industry's labor

[3] The author is grateful to Bert Balk for the reminder that the effects of changes in relative prices also sum to zero or vanish in TWD if the index formula is the Sato-Vartia, which is not CIA. This Sato-Vartia result of vanishing relative price effects in ALP growth can be seen in Equation (4) in Dumagan & Balk (2016).

employment remains constant, that is, $L_n^0 = L_n^1$ so that $L^0 = L^1$. Therefore, ALP growth in (14) yields GDP growth given by

$$\text{GDP growth} \equiv \frac{Y^1}{Y^0} - 1 = \sum_{n=1}^{N} s_{Y_n}^0 \left(\frac{Y_n^1}{Y_n^0} - 1\right) + \sum_{n=1}^{N} \left(\frac{p_n^1 Y_n^1}{Y^0} - s_{Y_n}^0 \frac{Y_n^1}{Y_n^0}\right) ; \quad (21)$$

$$\text{PGE (pure growth effect)} \equiv s_{Y_n}^0 \left(\frac{Y_n^1}{Y_n^0} - 1\right) ; \quad (22)$$

$$\text{PCE (price change effect)} \equiv \frac{p_n^1 Y_n^1}{Y^0} - s_{Y_n}^0 \frac{Y_n^1}{Y_n^0} . \quad (23)$$

It is important to note that although PCE in GDP growth assumes constant industry labor employment so that $L^0 = L^1$, $c_n^{p(0,1)}$ does not assume this but assumes constant industry labor productivities and labor shares in (16). Therefore, from (18), (19), and (23),

$$c_n^{p(0,1)} \equiv \frac{L^0}{L^1} \times \text{ PCE} = \frac{L^0}{L^1} \left(\frac{p_n^1 Y_n^1}{Y^0} - s_{Y_n}^0 \frac{Y_n^1}{Y_n^0}\right) ; \quad (24)$$

$$C^{p(0,1)} \equiv \frac{L^0}{L^1} \times \text{ Sum of PCE} = \frac{L^0}{L^1} \left(\frac{Y^1}{Y^0} - \sum_{n=1}^{N} s_{Y_n}^0 \frac{Y_n^1}{Y_n^0}\right) . \quad (25)$$

Dumagan (2018a) applied PGE in (22) and PCE in (23) to Italian value-added or GDP in chained prices based on the chained Laspeyres; Philippine GDP in constant prices based on direct Laspeyres; and also to the U.S. GDP in chained prices based on the Fisher index. Therefore, the appropriate $s_{Y_n}^0$ for Italy is the share of GDP in current prices and the appropriate $s_{Y_n}^0$ for the Philippines is the share of GDP in constant prices so that Sum of PCE $= 0$ and $C^{p(0,1)} = 0$ in both cases, according to (20). The results by Dumagan (2018a) are reproduced later in this paper.

In the case of the U.S. GDP, the appropriate $s_{Y_n}^0$ is the Fisher weight from the additive decomposition. The available data permit computing shares of GDP in current prices but these are inappropriate and if used, Sum of PCE $\neq 0$. To illustrate this result, PGE in (22) and PCE in (23) were applied to the U.S. GDP, 2009–2010, in Table 1 where Sum of PCE $= 0.35$. Total labor employment (in thousands) was 131,709 in 2009 and fell to 130,716 in 2010. Substituting these employment values and Sum of PCE $= 0.35$ into (25) yields $C^{p(0,1)} = 0.35$ percentage points, which would be the contribution to the U.S. ALP growth of 3.31% of changes in industry relative prices if all industry labor productivities and all labor shares are constant. In Table 1, $C^{p(0,1)} = 0.35$ is labeled as PCE Contribution to ALP Growth that is, however, erroneous because Sum of PCE $= 0.35$ used shares of GDP in current prices that are inappropriate for the U.S. GDP in chained prices based on the Fisher index.

Table 1
Growth of GDP and Aggregate Labour Productivity (ALP) in the United States, 2009-2010

	GDP in current prices		GDP in chained prices		GDP Growth Contributions		GDP Growth
					PGE	PCE	PGE + PCE
	(billions of dollars)		(billions of 2009 dollars)		(percentage points)		
	2009	2010	2009	2010	2010	2010	2010
Agriculture, forestry, fishing, and hunting	137.7	160.2	137.7	140.3	0.0180	0.1246	0.1426
Mining	290.3	331.7	290.3	272.7	-0.1221	0.3814	0.2594
Utilities	250.8	267.0	250.8	274.4	0.1637	-0.0737	0.0900
Construction	577.3	541.6	577.3	551.6	-0.1782	-0.1147	-0.2930
Manufacturing	1726.7	1830.6	1726.7	1818.2	0.6346	-0.0673	0.5673
Wholesale trade	822.8	868.5	822.8	848.3	0.1769	0.0674	0.2442
Retail trade	842.1	868.8	842.1	862.1	0.1387	-0.0263	0.1124
Transportation and warehousing	398.8	425.1	398.8	421.4	0.1567	-0.0099	0.1468
Information	705.3	730.2	705.3	735.1	0.2067	-0.0951	0.1115
Finance, insurance, real estate, rental, and leasing	2874.1	2951.6	2874.1	2925.4	0.3558	-0.0655	0.2903
Professional and business services	1661.1	1729.7	1661.1	1718.0	0.3946	-0.0637	0.3309
Educational services, health care, and social assistance	1214.0	1248.5	1214.0	1220.5	0.0451	0.0896	0.1347
Arts, entertainment, recreation, accommodation, and food services	522.3	540.7	522.3	541.3	0.1318	-0.0494	0.0823
Other services, except government	329.5	332.4	329.5	323.9	-0.0388	0.0311	-0.0077
Government	2065.8	2137.9	2065.8	2079.8	0.0971	0.2239	0.3210
Sum of Industry Contributions to GDP Level and Growth	**14418.6**	**14964.5**	**14418.6**	**14783.8**	**2.1805**	**0.3523**	**2.5328**
Total Labour Employment (thousands)			**131709.0**	**130716.0**			
ALP Growth				**3.31**			
PCE Contribution to ALP Growth (assuming industry labour productivities and labour shares are all constant)						**0.35**	

Source: GDP and employment data are from the US Bureau of Economic Analysis. PGE, PCE, and PCE Contribution to ALP Growth (last row) are the author's calculations based on formulas in the text according to the TWD growth decomposition where the weights are shares of GDP in current prices.

As noted earlier, the Fisher index underlying U.S. GDP is not CIA. Therefore, industry contributions to GDP growth are sensitive to the level of aggregation. This means, for example, that if the first sector of *Agriculture, forestry, fishing, and hunting* (Table 1) is disaggregated into the component industries and the individual PGE and PCE of these industries are computed, the sum of the individual PGE's will not necessarily equal the sector's PGE of 0.02 percentage points in row 1. Similarly, the sum of the individual PCEs will not necessarily equal the sector's PCE of 0.12 percentage points. Therefore, the Sum of PCE = 0.35 could change with disaggregation of sectors. This is shown in Appendix Table 1 where the 15 sectors in Table 1 were disaggregated into 71 industries that appear to be the most detailed GDP disaggregation downloadable from the U.S. BEA website (https://www.bea.gov). As a result, the erroneous Sum of PCE went down to 0.27 percentage points and, given the same total employment data, $C^{p(0,1)}$ also went down to 0.27 percentage points. This shows that the erroneous contribution of changes in relative prices, in Diewert's special case of constant industry labor productivities and labor shares, is not negligible because 0.27 amounts to over 8% of ALP growth of 3.31%.

The result above that $C^{p(0,1)} \neq 0$ will also follow from Diewert's (2015) TWD application to Australia because $s_{Y_n}^0$ in his formula in (5) is the share of GDP in current prices but Australian value added or GDP in his study is in constant 1995 dollars. However, $C^{p(0,1)} = 0$ will hold in Dumagan's (2013) TWD application to Italy where GDP is based on chained Laspeyres but not in the application to Thailand where GDP is in constant prices because $s_{Y_n}^0$ in both applications is the share of GDP in current prices. Moreover, $C^{p(0,1)} \neq 0$ will result from Dumagan's (2013) application to the United States and from Tang and Wang's (2004, 2015) applications to Canada and the United States where GDP is based on chained Fisher because these applications employed ALP growth in (14) and GDP growth in (21) where $s_{Y_n}^0$ is share of GDP in current prices. Therefore, $C^{p(0,1)} \neq 0$ in Table 1 and Appendix Table 1 illustrate their erroneous results. This finding is especially ironic for Tang and Wang because they originated TWD and first applied it to Canada and the United States.

Diewert's Apparent Concession of Zero Aggregate Price Effects

Diewert (2016) encountered the puzzling result (i.e., contrary to his claim in Diewert, 2015) that "the effects of real output price change, when aggregated over industries, are insignificant" (p. 5) and proceeded to explain this puzzle as follows.

Recall from (1) the aggregate price index P^t and the industry price indexes P_n^t. By first-order Taylor series approximation, Diewert (2016) showed that the growth of the aggregate price index yields[4]

$$\ln\left(\frac{P^1}{P^0}\right) \approx \sum_{n=1}^{N} s_{y_n}^0 \ln\left(\frac{P_n^1}{P_n^0}\right). \qquad (26)$$

Subtracting the left-hand side from both sides, using the fact that $\sum_{n=1}^{N} s_{y_n}^0 = 1$, it follows that

$$0 \approx \sum_{n=1}^{N} s_{y_n}^0 \ln\left(\frac{P_n^1}{P_n^0}\right) - \sum_{n=1}^{N} s_{y_n}^0 \ln\left(\frac{P^1}{P^0}\right) = \sum_{n=1}^{N} s_{y_n}^0 \ln\left(\frac{P_n^1/P^1}{P_n^0/P^0}\right) = \sum_{n=1}^{N} s_{y_n}^0 \ln\left(\frac{p_n^1}{p_n^0}\right). \qquad (27)$$

Diewert's approximation result in (27) is supported by this paper's results in (17) and (20) that given the "appropriate" $s_{Y_n}^0$, price change effects will sum to zero. That is,

$$C^{p(0,1)} \equiv \sum_{n=1}^{N} s_{Y_n}^0 \left(\frac{p_n^1}{p_n^0} - 1\right)\frac{Y_n^1}{Y_n^0}\frac{L^0}{L^1} = \frac{L^0}{L^1}\left(\frac{Y^1}{Y^0} - \sum_{n=1}^{N} s_{Y_n}^0 \frac{Y_n^1}{Y_n^0}\right) = 0\,. \qquad (28)$$

Recall that (28) is this paper's aggregate effect of changes in relative prices in Diewert's special case where industry labor productivities and labor shares are all constant, in which case $(Y_n^1/Y_n^0)(L^0/L^1) = 1$. Moreover, because price indexes are strictly positive, their *arithmetic* growth rates cannot be smaller than their corresponding *logarithmic* growth rates (Dumagan & Ball, 2009). That is, $[(p_n^1/p_n^0) - 1] \geq \ln(p_n^1/p_n^0)$. Therefore, (27) and (28) yield

[4] The author is grateful to Erwin Diewert for pointing out the approximations in (26) and (27) from Diewert (2016), which came out earlier as Discussion Paper 14-10, November 2, 2014, School of Economics, University of British Columbia, Vancouver, B.C., Canada, V6T 1Z1.

$$\sum_{n=1}^{N} s_{y_n}^{0} \ln\left(\frac{p_n^1}{p_n^0}\right) \approx \sum_{n=1}^{N} s_{Y_n}^{0} \left(\frac{p_n^1}{p_n^0} - 1\right) \frac{Y_n^1}{Y_n^0} \frac{L^0}{L^1} = \frac{L^0}{L^1}\left(\frac{Y^1}{Y^0} - \sum_{n=1}^{N} s_{Y_n}^{0} \frac{Y_n^1}{Y_n^0}\right) = 0\,. \tag{29}$$

The approximation applies because Diewert's $s_{y_n}^0$ in the left-hand side of (29) is the share of GDP in current prices whereas this paper's $s_{y_n}^0$ in the right-hand side could be different, depending on the GDP quantity index. It could be the share of GDP in current prices for chained Laspeyres; share of GDP in constant prices for direct Laspeyres; and Fisher weight for chained Fisher.

Based on (29), Diewert's (2016) results from Australian data in constant 1995 dollars—that price change effects did not matter much when aggregated across industries—does not anymore appear puzzling. Considering that shares in current prices do not differ much from those in constant prices, especially for years close to the base year, the sum of price effects should not differ much from zero. However, it is important to point out that when using the inappropriate shares of GDP in current prices, even the right-hand side of (29) could yield a sum of price change effects (Sum of PCE) different from zero and the difference is sensitive to the level of aggregation although the direct Laspeyres quantity index underlying GDP in constant prices is CIA. An example is shown in Dumagan (2017, Table 6, p. 19) by a GEAD decomposition, which is the same as TWD, of growth of Philippine GDP in constant prices (2009–2010) where Sum of PCE $= 0.02$ percentage points if the Services sector is treated as one whole but switches to Sum of PCE $= -0.06$ if this sector is disaggregated into the component industries. However, these results are "close" to zero although the shares of GDP in current prices used as weights are inappropriate.

Tang and Wang's Insistence on Non-zero Aggregate Price Effects: A Disproof

Tang and Wang insisted that:

> We believe that aggregate GDP growth is NOT pure quantity growth because aggregate real GDP, in the chain-Fisher index, has price effect. Under the chain-Fisher world, not only quantity changes but also price changes. So . . . changes in both quantity and price affect aggregate real GDP. (personal communications, September 14, 2018) [5]

At this point, there is no disagreement between this paper and Tang and Wang about the existence of price effects on industry contributions to GDP growth. The dispute arises from their belief that if the GDP index is chained Fisher, the aggregate price effect is non-zero, contrary to this paper's finding in (29) that it is zero given the appropriate replacement for $s_{Y_n}^0$, p_n^0, p_n^1, Y_n^0, and Y_n^1 as explained below. However, they are silent on the aggregate price effect when the GDP index is not chained Fisher, for example, direct Laspeyres in the Philippines or chained Laspeyres in Italy which are illustrated empirically later in this paper.

Indeed, in Tang and Wang's (2004, 2015) growth decompositions in Canada and the United States where GDP is based on chained Fisher, (29) can be shown not equal to zero—similar to Sum of PCE $= 0.35$ (Table 1) and Sum of PCE $= 0.27$ (Appendix Table 1)[6]—and, therefore, they believed as stated above that "aggregate GDP growth is NOT pure quantity growth because aggregate real GDP, in the chain-Fisher index, has price effect" (personal communications, September 14, 2018). However, their belief is ill-founded because $s_{Y_n}^0$, p_n^0, p_n^1, Y_n^0, and Y_n^1 as defined in (29) are not appropriate for GDP based on the Fisher index.

[5] I sent an earlier draft of this paper and a copy of Dumagan (2018a) to Jianmin Tang on August 20, 2018, and also a revised draft on August 31, 2018. The above quote is from Jianmin Tang's reply together with a copy of Tang and Wang (2014) on September 14, 2018, for which the author is grateful.

[6] Note from (25) that the formula for Sum of PCE is the right-most term in (29) inside the parenthesis. For the US, Dumagan (2017) found that depending on the level of aggregation, Sum of PCE could switch signs ranging from -0.22 to 0.35 during 1997-2015 where 0.35 happened during 2009-2010 (Table 1 in this paper).

The appropriate $s_{Y_n}^0$ is the Fisher weight from the additive decomposition. However, detailed data to compute the "true" Fisher weights of the price relatives and quantity relatives of the aggregate Fisher price and quantity indexes are not available. However, published data on U.S. GDP in current prices and chained prices may be used to compute Fisher weights and show the Sum of PCE $= 0$ for "illustrative" purposes only. In this illustration, the implicit deflators of industries may be treated like "prices" and their GDPs in chained prices may be treated like "quantities" to implement the additive decomposition formula for the growth of the Fisher quantity index denoted below by $Q^F - 1$. For the above purposes, this formula (Balk, 2004; Dumagan, 2002; Moulton & Seskin, 1999; Van IJzeren, 1952) may be expressed as:

$$\text{Level} \equiv Q^F = \sum_{n=1}^{N} S_{Y_n}^F \frac{Y_n^1}{Y_n^0} \quad ; \quad \text{Growth} \equiv Q^F - 1 = \sum_{n=1}^{N} S_{Y_n}^F \left(\frac{Y_n^1}{Y_n^0} - 1 \right); \tag{30}$$

$$\text{Fisher quantity index weight} \equiv S_{Y_n}^F = \left[\left(\frac{P^F}{P^L + P^F} \right) S_{Y_n}^L + \left(\frac{P^L}{P^L + P^F} \right) S_{Y_n}^P \right]; \tag{31}$$

$$\text{Fisher price index} \equiv P^F \quad ; \quad \text{Laspeyres price index} \equiv P^L ; \tag{32}$$

$$\text{Laspeyres quantity index weight} \equiv S_{Y_n}^L = \frac{P_n^0 Y_n^0}{\sum_{n=1}^{N} P_n^0 Y_n^0} = S_{Y_n}^0 \text{, as defined in (5)} ; \tag{33}$$

$$\text{Paasche quantity index weight} \equiv S_{Y_n}^P = \frac{P_n^1 Y_n^0}{\sum_{n=1}^{N} P_n^1 Y_n^0} . \tag{34}$$

In (30) to (34), P_n^0 and P_n^1 are industry implicit deflators acting like prices and Y_n^0 and Y_n^1 are industry GDP in chained prices acting like quantities for illustrative purposes. As such, they yield the price indexes as well as the weights defined above to obtain the growth of the Fisher quantity index, $Q^F - 1$, and the contributions of each industry in (30).

For comparison with the results from TWD in Table 1, the Fisher additive decomposition is applied to the same data and the results are shown in Table 2. Hence, in decomposing U.S.

GDP growth into industry PGE in (22) and PCE in (23), the only source of difference is that TWD uses as weights the shares of GDP in current prices, $S_{Y_n}^0$ in (5), which equals $S_{Y_n}^L$ as shown in (33), whereas the Fisher additive decomposition uses the Fisher weights, $S_{Y_n}^F$ in (31). The comparison is illustrated for U.S. GDP during 2009–2010, which is convenient because 2009 is the base year.

The TWD results in Table 1 show the U.S. GDP (billion chained 2009 dollars) from BEA of 14,418.6 in 2009 and 14,783.8 in 2010 which yield a growth of 2.53%. In contrast, the Fisher additive decomposition results in Table 2 imply that the U.S. GDP (billion chained 2009 dollars) is the same 14,418.6 in 2009, but GDP is 14,418.6 × (1.0216) = 14,730.1 in 2010 which yields a lower growth of 2.16%. The difference, as explained earlier, is that this paper's aggregate Fisher quantity index of 1.0216 is computed from data that differ in detail from that used by BEA to calculate the true aggregate Fisher indexes. Moreover, the 15 sectors (Table 2) were disaggregated into 71 industries with similar results (Appendix Table 2) that
$\text{Growth of Fisher Quantity Index} = \text{Sum of PGE} = \text{GDP Growth and Sum of PCE} = 0$, except for a slight increase in the aggregate Fisher index from 1.0216 to 1.0217. Also, each sector's contribution to the growth of the Fisher quantity index exactly equals the sector's PGE, the quantity effect on GDP growth. Therefore, GDP growth is pure quantity growth, contrary to Tang and Wang's belief.

The illustration suffices to show that although TWD yields Sum of PCE $\neq 0$, for example, 0.35 percentage points (Table 1) or 0.27 percentage points (Appendix Table 1), the Fisher additive decomposition yields Sum of PCE $= 0$ (both in Table 2 & Appendix Table 2), implying that relative price changes have no effect on GDP growth in the aggregate. Moreover, (25) implies that $C^{p(0,1)} = (L^0/L^1) \times \text{Sum of PCE} = 0$, if labor productivities and labor shares are all constant. Therefore, the $C^{p(0,1)} = 0.35$ percentage points contribution to ALP growth by TWD in Table 1 will vanish if Fisher weights are used in place of GDP shares in current prices.

Table 2
Growth of Aggregate Fisher Quantity Index and GDP in the United States, 2009-2010

	Aggregate Fisher Quantity Index			GDP Growth Contributions (percentage points)		GDP Growth
	Weights	Level	Growth	PGE	PCE	PGE + PCE
	2010	2010	2010	2010	2010	2010
Agriculture, forestry, fishing, and hunting	0.0101	0.0103	0.0191	0.0191	0.0006	0.0198
Mining	0.0221	0.0208	-0.1341	-0.1341	0.0019	-0.1322
Utilities	0.0170	0.0186	0.1602	0.1602	-0.0004	0.1598
Construction	0.0394	0.0376	-0.1752	-0.1752	-0.0006	-0.1759
Manufacturing	0.1192	0.1255	0.6317	0.6317	-0.0006	0.6311
Wholesale trade	0.0573	0.0591	0.1775	0.1775	0.0002	0.1778
Retail trade	0.0582	0.0595	0.1381	0.1381	-0.0002	0.1379
Transportation and warehousing	0.0276	0.0291	0.1562	0.1562	-0.0001	0.1561
Information	0.0484	0.0504	0.2044	0.2044	-0.0006	0.2038
Finance, insurance, real estate, rental, and leasing	0.1986	0.2022	0.3545	0.3545	-0.0007	0.3539
Professional and business services	0.1147	0.1186	0.3928	0.3928	-0.0005	0.3923
Educational services, health care, and social assistance	0.0845	0.0849	0.0452	0.0452	0.0003	0.0455
Arts, entertainment, recreation, accommodation, and food services	0.0359	0.0372	0.1307	0.1307	-0.0003	0.1303
Other services, except government	0.0230	0.0226	-0.0390	-0.0390	0.0001	-0.0389
Government	0.1441	0.1451	0.0977	0.0977	0.0009	0.0985
Sum of Industry Contributions	**1.0000**	**1.0216**	**2.1598**	**2.1598**	**0.0000**	**2.1598**

Source: Author's calculations based on the same GDP data in Table 1 and formulas in the text where the industry shares of GDP in current prices used by TWD are replaced by Fisher weights above from the Fisher additive decomposition.

It may be noted (Table 2 & Appendix Table 2) that PCE for each industry or sector in 2010 is very small, rounding off to zero in two decimal places (i.e., 0.00). It may be suspected that this could be because 2009 is the base year so that the implicit deflators used as prices are all equal to "1" in 2009. It turns out that this fact is immaterial and the result that each PCE rounds off to "0.00" is typical and can be verified by similar calculations each year from 2010 to 2014.[7] This typical result holds true for PCE of each sector—either in Table 2 with 15 sectors or in Appendix Table 2 with 71—and Sum of PCE = 0 holds up to 15 decimal places, thus, "truly

[7] This result should not appear surprising under chaining. Recall that relative price is a ratio of an industry's GDP deflator to the economy's GDP deflator. Therefore, in the US, relative prices are ratios of chained price indexes. As such, the "quantity" weights of these price indexes are updated at the same time from one period to the next and, thus, the changes of these ratios will tend to be small, especially in years close to the base year 2009 when they are all unitary. This explains why each industry's PCE rounded out to 0.00 as noted above from 2010 to 2014.

zero" each year during 2010–2014. This finding disproves Tang and Wang's belief that the Sum of PCE $\neq 0$.

To be fair, there are price changes embodied in the Fisher quantity index being the geometric mean of the Laspeyres and Paasche quantity indexes and considering that the latter indexes have different price weights. The embodied price changes may be inferred from the formula for the Fisher weight, $S_{Y_n}^F$, in (31) which depends on the Laspeyres and Fisher price indexes. To some extent, this may explain why PCE could differ from zero (Table 2 & Appendix Table 2), although each PCE rounds off to 0.00 as noted above. Therefore, the result that the Sum of PCE $= 0$ means that there are no residual price change effects on GDP growth even in the case of the Fisher quantity index that underlies GDP in Canada and the United States. Moreover, this conclusion is unaffected by price chaining as explained below.

In general, a quantity index chained from base period 0 to some distant period t+1 may be defined as $Q^{0,t+1} \equiv Q^{0,0} \times Q^{0,1} \times Q^{1,2} \times \cdots \times Q^{t-1,t} \times Q^{t,t+1} = Q^{0,t} \times Q^{t,t+1}$ where $Q^{0,0} = 1$. Therefore, if Y^0 is GDP in base period 0, GDP in chained prices is $Y^t = Y^0 \times Q^{0,t}$ at t and $Y^{t+1} = Y^0 \times Q^{0,t+1}$ at t+1. Hence, GDP growth is $(Y^{t+1}/Y^t) - 1 = (Q^{0,t+1}/Q^{0,t}) - 1 = Q^{t,t+1} - 1$. The last result implies that the accumulated effects of price chaining by updating the price weights from 0 to t cancels out in GDP growth from t to t+1. That is, from t to t+1, GDP growth consists only of quantity growth (i.e., no residual price effects). Hence, the Sum of PCE $= 0$ also holds for a chained GDP quantity index given the appropriate weights like $S_{Y_n}^F$ in (31) for the Fisher.

Finally, it may be remarked that their belief that Sum of PCE $\neq 0$ could be due to Tang and Wang's apparent lack of recognition of the limited role of relative price, $p_n^t \equiv P_n^t/P^t$, in their framework. Consider from (1) that P^t determines aggregate real GDP, Y^t, by deflating (or dividing) the corresponding aggregate nominal GDP. Similarly, P_n^t determines industry real GDP, Y_n^t, by deflating the corresponding industry nominal GDP. Thus, $p_n^t \equiv P_n^t / P^t$ has nothing to do with the levels of Y^t and Y_n^t. However, (1) shows that the additivity of nominal GDP implies that aggregate real GDP may be expressed as $Y^t = \sum_{n=1}^{N} p_n^t Y_n^t$ where p_n^t converts Y_n^t to the same unit as Y^t. That is, p_n^t only changes the valuation of Y_n^t for additivity and, therefore, changes the composition of Y^t but without affecting the predetermined level of Y^t. In this light, changes in p_n^t do not change the level and, therefore, do not affect the growth of Y^t.

It appears that relative price performs only a "valuation" function to homogenize each industry's real GDP to the same units as the economy's real GDP. It cannot perform its theoretical "allocation" function because the existing TWD framework is a growth "accounting" procedure where relative price has no functional relation to labor share and, therefore, has no claim to the growth effects of changes in labor shares.

Relative Price Change Effects in the General Case: Empirical Applications

It may now be shown that if the weights of industry contributions are appropriate, the effects of price changes still sum to zero in the TWD decomposition in the general case where industry labor productivities and labor shares change. Note from (11) to (13) that by definition, an industry's contribution to ALP growth is the sum of WSPGE and ISRE which is

$$\text{WSPGE} + \text{ISRE} = s_{Y_n}^0 \left(\frac{Y_n^1 / L_n^1}{Y_n^0 / L_n^0} - 1 \right) + \left(\frac{p_n^1 Y_n^1 / L^1}{Y^0 / L^0} - s_{Y_n}^0 \frac{Y_n^1 / L_n^1}{Y_n^0 / L_n^0} \right). \tag{35}$$

Relative price, p_n^1, appears in ISRE but disappears in the sum of (35) as shown by

$$\text{Sum of WSPGE} + \text{Sum of ISRE} = \sum_{n=1}^{N} s_{Y_n}^0 \left(\frac{Y_n^1 / L_n^1}{Y_n^0 / L_n^0} - 1 \right) + \left(\frac{Y^1 / L^1}{Y^0 / L^0} - \sum_{n=1}^{N} s_{Y_n}^0 \frac{Y_n^1 / L_n^1}{Y_n^0 / L_n^0} \right). \tag{36}$$

The result in (36) shows that relative prices do not appear in overall ALP growth and, thus, implies that the effects of relative price changes sum to zero or totally cancel out. Therefore, ALP growth depends only on changes in industry labor productivities and labor shares.

Finally, the traditional (TRAD) decomposition of ALP growth may be compared to TWD by applying them to GDP in constant prices. This comparison is enlightening because TRAD is applicable only to GDP in constant prices whereas TWD is applicable to any GDP. In this application, TWD yields TRAD as a special case when relative prices are set to "1" which is permissible because GDP in constant prices is additive without relative prices. That is, (1) yields

$$\text{GDP in constant prices in TWD} \equiv Y^t = \sum_{n=1}^{N} p_n^t Y_n^t \; ; \tag{37}$$

$$\text{GDP in constant prices in TRAD} \equiv Y^t = \sum_{n=1}^{N} Y_n^t \; . \tag{38}$$

The direct Laspeyres quantity index, which is CIA, underlies GDP in constant prices. In this case, the CIA property of this index means that the appropriate $s_{Y_n}^0$ are shares of GDP in constant prices, which are $s_{Y_n}^0 = Y_n^0/Y^0$. Therefore, (35) and (36) yield for the same industry

$$\text{WSPGE in TWD} = \text{WSPGE in TRAD} \equiv \frac{Y_n^0}{Y^0}\left(\frac{Y_n^1/L_n^1}{Y_n^0/L_n^0} - 1\right) \; ; \tag{39}$$

$$\begin{aligned} \text{ISRE in TWD} &\equiv \left[\frac{p_n^1 Y_n^1/L^1}{Y^0/L^0} - \left(\frac{Y_n^0}{Y^0}\right)\frac{Y_n^1/L_n^1}{Y_n^0/L_n^0}\right] \\ &\neq \text{ISRE in TRAD} \equiv \left[\frac{Y_n^1/L^1}{Y^0/L^0} - \left(\frac{Y_n^0}{Y^0}\right)\frac{Y_n^1/L_n^1}{Y_n^0/L_n^0}\right] \; . \end{aligned} \tag{40}$$

The inequality in (40) is due to relative price, p_n^1, that is accounted for by TWD but ignored by TRAD in the contribution of an industry. However, (37) and (38) imply that (40) yields, Sum of ISRE in TWD = Sum of ISRE in TRAD

$$= \frac{Y^1/L^1}{Y^0/L^0} - \sum_{n=1}^{N} \left(\frac{Y_n^0}{Y^0}\right) \frac{Y_n^1/L_n^1}{Y_n^0/L_n^0} \neq 0 \text{ if } \frac{L^1}{L^0} \neq \frac{L_n^1}{L_n^0} \text{ or } \frac{L_n^0}{L^0} \neq \frac{L_n^1}{L^1} \;; \quad (41)$$

Sum of ISRE in TWD = Sum of ISRE in TRAD

$$= \frac{L^0}{L^1}\left[\frac{Y^1}{Y^0} - \frac{Y^1}{Y^0}\right] = 0 \quad \text{if} \quad \frac{L^1}{L^0} = \frac{L_n^1}{L_n^0} \quad \text{or} \quad \frac{L_n^0}{L^0} = \frac{L_n^1}{L^1} \,. \qquad (42)$$

Given the inequality in (40) due to the absence of relative prices in TRAD, the equalities that Sum of ISRE in TWD = Sum of ISRE in TRAD in (41) and (42) imply that relative price effects cancel out in TWD. Therefore, if the appropriate weights are used, TWD and TRAD yield the same result that ALP growth depends only on changes in labor productivities and labor shares.

The results in (39) and (41) are borne out by the application in Dumagan (2018a) to the Philippines, 2009–2010, where GDP is in constant prices. To avoid any confusion, the TWD framework in this paper is exactly the same as the GEAD framework in Dumagan (2018a) where Table 6 (p. 23) is reproduced below as Table 3.

In Table 3, for each industry, WSPGE in TWD = WSPGE in TRAD according to (39) but ISRE differs between TWD and TRAD according to (40) because TRAD ignores relative prices. However, for all industries, the Sum of ISRE in TWD = Sum of ISRE in TRAD = 1.53 percentage points based on (41). This equality implies that the aggregate relative price effect on ALP growth in TWD is zero for all industries.

Moreover, the choice of shares of GDP as weights in Table 3 is crucial because it could reverse the sign of Sum of ISRE as shown in Table 4 where in 2011, the Sum of ISRE was -0.0302 percentage points if the appropriate shares of GDP in constant prices are used but will change sign to 0.1012 if the inappropriate shares of GDP in current prices are used instead.

Table 3
ALP Growth in the Philippines, 2009-2010

	WSPGE		ISRE		WSPGE + ISRE	
	TRAD	TWD	TRAD	TWD	TRAD	TWD
Growth contributions by industry			(Percentage points)			
Agriculture, Fishery & Forestry	0.0704	0.0704	-0.4298	0.2945	-0.3594	0.3649
Mining and Quarrying	-0.0785	-0.0785	0.1726	0.4594	0.0941	0.3809
Manufacturing	1.2993	1.2993	0.4509	-0.3175	1.7503	0.9818
Construction	0.3865	0.3865	0.2175	0.6446	0.6040	1.0310
Electricity, Gas and Water Supply	0.1478	0.1478	0.0943	0.1006	0.2420	0.2484
Transport, Storage and Communications	-0.0473	-0.0473	-0.0891	-1.1278	-0.1364	-1.1750
Trade	0.6204	0.6204	0.2746	1.0375	0.8950	1.6579
Financial Intermediation	0.1054	0.1054	0.3521	0.7089	0.4574	0.8142
Other services	0.6836	0.6836	0.4844	-0.2729	1.1680	0.4107
Sum	3.1875	3.1875	1.5275	1.5275	4.7150	4.7150
ALP Growth					4.7150	4.7150

Source: Author's calculations based on formulas in this paper applied to GDP and employment data (Table 4, p. 22, Dumagan (2018a)) from the Philippine Statistics Authority. Table 3 above is the same as Table 6, p. 23 in Dumagan (2018a) where the Tang-Wang-Diewert (TWD) decomposition was originally called by Dumagan (2013, 2018a) as a generalized exactly additive decomposition (GEAD). The difference, however, is that GEAD or TWD in this paper was modified to generate the results in this table using--as weights of industry ALP growth contributions--the shares of GDP in constant prices that are "appropriate" for the *direct* Laspeyres quantity index underlying GDP in constant prices in countries like the Philippines.

Table 4
ALP Growth in the Philippines, Selected Years (2002-2011)

	2002	**2005**	**2008**	**2011**
TWD: ISRE by industry		(Percentage points)		
Agriculture, Fishery & Forestry	-0.9394	-0.6717	0.5535	1.2170
Mining and Quarrying	0.0937	0.2784	0.3136	0.3134
Manufacturing	-0.6624	-0.0577	-1.4614	-1.6918
Construction	0.2120	0.2463	0.4267	0.4292
Electricity, Gas and Water Supply	0.0662	0.0695	-0.3637	-0.2184
Transport, Storage and Communications	-0.0194	-0.3468	-1.1702	-1.1870
Trade	0.1661	0.2778	0.5466	1.1962
Financial Intermediation	0.1874	0.2392	0.3788	0.7023
Other services	0.3564	-0.3936	-0.3271	-0.7910
TWD: Sum of ISRE (correct shares)	**-0.5395**	**-0.3586**	**-1.1032**	**-0.0302**
TWD: Sum of ISRE (incorrect shares)	**-0.5651**	**-0.3784**	**-1.0692**	**0.1012**
TWD: Sum of WSPGE	**1.0600**	**2.8729**	**3.6374**	**0.7464**
TWD: ALP Growth = Sum of WSPGE + Sum of ISRE	**0.5205**	**2.5143**	**2.5342**	**0.7162**

Source: Author's calculations based on formulas in this presentation applied to GDP and employment data in Table 3.

Note that a negative Sum of ISRE could be interpreted to mean that ALP growth is lower than it could have been if labor were properly allocated (i.e., moving from industries with productivities lower than ALP to those with productivities higher than ALP). Thus, the sign reversal from the use of inappropriate shares of GDP in current prices could lead to analytic misinterpretations and misdirected policies for promoting ALP growth.

Finally, recall that GDP growth is a special case of ALP growth when each industry's labor employment remains constant, that is, $L_n^0 = L_n^1$ so that $L^0 = L^1$. In this case, recalling (21), (22), and (23), WSPGE and ISRE in ALP growth become, respectively, the pure growth effect (PGE) and the price change effect (PCE) in GDP growth. Therefore, the ALP growth decomposition in (39) to (42) yield the corresponding GDP growth decomposition given by

$$\text{PGE in TWD} = \text{PGE in TRAD} \equiv \frac{Y_n^0}{Y^0}\left(\frac{Y_n^1}{Y_n^0} - 1\right) ; \tag{43}$$

$$\text{PCE in TWD} \equiv \left[\frac{p_n^1 Y_n^1}{Y^0} - \frac{Y_n^1}{Y^0}\right] \neq \text{PCE in TRAD} \equiv \left[\frac{Y_n^1}{Y^0} - \frac{Y_n^1}{Y^0}\right] = 0\,. \tag{44}$$

The inequality in (44) is due to relative price, p_n^1, that is accounted for by TWD but ignored by TRAD in the contribution of an industry. However, additivity of GDP in constant prices in (37) and (38) imply that (43) and (44) yield

$$\begin{aligned}\text{Sum of PGE in TWD} = \text{Sum of PGE in TRAD} &= \sum_{n=1}^{N} \frac{Y_n^0}{Y^0}\left(\frac{Y_n^1}{Y_n^0} - 1\right) \\ &= \frac{Y^1}{Y^0} - 1 = \text{GDP growth}\,;\end{aligned} \tag{45}$$

$$\begin{aligned}\text{Sum of PCE in TWD} = \text{Sum of PCE in TRAD} &= \sum_{n=1}^{N} \left[\frac{p_n^1 Y_n^1}{Y^0} - \frac{Y_n^1}{Y^0}\right] \\ &= \left[\frac{Y^1}{Y^0} - \frac{Y^1}{Y^0}\right] = 0\,.\end{aligned} \tag{46}$$

For illustration, (43) to (46) were applied to the same Philippine GDP data used in Table 3 and the results are shown in Table 5.

Table 5
GDP Growth in the Philippines, 2009-2010

	PGE		PCE		PGE + PCE	
	TRAD	TWD	TRAD	TWD	TRAD	TWD
Growth contributions by industry			(Percentage points)			
Agriculture, forestry, and fishery	-0.0204	-0.0204	0.0000	0.7446	-0.0204	0.7242
Mining and quarrying	0.1278	0.1278	0.0000	0.2949	0.1278	0.4226
Manufacturing	2.3973	2.3973	0.0000	-0.7898	2.3973	1.6074
Construction	0.7707	0.7707	0.0000	0.4389	0.7707	1.2096
Electricity, gas and water supply	0.3461	0.3461	0.0000	0.0065	0.3461	0.3526
Transport Communication and Storage	0.0825	0.0825	0.0000	-1.0676	0.0825	-0.9851
Trade	1.3805	1.3805	0.0000	0.7842	1.3805	2.1646
Finance	0.6492	0.6492	0.0000	0.3667	0.6492	1.0159
Other Services	1.8987	1.8987	0.0000	-0.7784	1.8987	1.1203
Sum	**7.6323**	**7.6323**	**0.0000**	**0.0000**	**7.6323**	**7.6323**
GDP Growth					**7.6323**	**7.6323**

Source: Author's calculations based on procedures and data as explained in Table 3 above. This Table 5 is the same as Table 5, p. 22 in Dumagan (2018a) except for the change in acronym from GEAD to TWD.

The GDP growth of 7.6% is pure quantity growth as it is the sum of the PGE contributions of all industries that depend only on their real GDP growth. For each industry, PGE is the same in TRAD and TWD. Relative prices are ignored in TRAD but recognized in TWD. Thus, the PCE is zero for each industry in TRAD but non-zero in TWD. However, in TWD, the Sum of PCE is zero. This means that PCE matters only for individual industries but not for the whole economy. Therefore, as in the case of ALP growth (Table 3), there are no residual price change effects in GDP growth (Table 5).

Up to this point, TWD with the appropriate weights was applied to the United States where GDP is in chained prices based on the chained Fisher quantity index and to the Philippines where GDP is in constant prices based on the direct Laspeyres quantity index. In current practice, the only other GDP in chained prices based on the chained Laspeyres quantity index is Italy, for example, and in other countries in the European Union.

Therefore, to complete the worldwide illustration of the TWD decomposition when using the appropriate weights, TWD may be applied to Italian value-added (GDP) and employment data. In this case, the TWD decomposition of industry contributions to ALP growth—given by

WSPGE and ISRE in (35) and (36)—applies and the appropriate $s^0_{Y_n}$ weights are GDP shares in current prices as defined in (5) because the underlying GDP quantity index is the chained Laspeyres (Dumagan, 2018a). The results are presented in Table 6.

Table 6
ALP Growth in Italy, 2009-2010

	TWD		TWD
	WSPGE	**ISRE**	**WSPGE + ISRE**
Growth contributions by industry	(Percentage points)		
Agriculture, hunting and forestry; fishing and operation of fish hatcheries and fish farms	-0.0120	0.0630	0.0510
Industry, including energy	1.6836	-0.9253	0.7583
Construction	-0.1275	0.0260	-0.1015
Wholesale and retail trade, repair of motor vehicles and household goods, hotels and restaurants; transport and communication	0.7253	-0.3121	0.4132
Financial, real estate, renting and business activities	0.0076	0.5535	0.5611
Other service activities (by category)	-0.0018	0.5482	0.5464
Sum	**2.2752**	**-0.0468**	**2.2284**
ALP growth			**2.2284**

Source: Author's calculations based on formulas in this paper applied to value-added (GDP) and employment data (Table 1, p. 16, Dumagan (2018a)) from Istat - Istituto Nazionale di Statistica. Table 6 above is the same as Table 3, p. 18 in Dumagan (2018a) where the Tang-Wang-Diewert (TWD) decomposition was originally called by Dumagan (2013, 2018a) as a generalized exactly additive decomposition (GEAD). The difference, however, is that GEAD or TWD in this paper was modified to generate the results in this table using--as weights of industry ALP growth contributions--the shares of GDP in current prices that are "appropriate" for the *chained* Laspeyres quantity index underlying GDP in chained prices in countries like Italy.

In the TWD framework, ISRE for each industry combines the effects of changes in both labor shares and relative prices, but the Sum of ISRE has no residual price effects if the appropriate weights are used as shown in (36). Therefore, there are no price effects in ALP growth because WSPGE excludes price effects by definition.

As already noted, GDP growth is a special case of ALP growth when labor employment remains the same in each industry (i.e., labor shares also remain the same). In the TWD terminology in this paper, WSPGE in ALP growth becomes PGE in GDP growth whereas ISRE in ALP growth becomes PCE in GDP growth. As labor shares remain the same in GDP growth, the result that the Sum of ISRE in ALP growth has no residual price change effects necessarily implies that the Sum of PCE is zero for all industries. This is shown in the value-added (GDP) growth decomposition for Italy in Table 7.

Table 7
Value-Added Growth in Italy, 2009-2010

		TWD	TWD
	PGE	**PCE**	**PGE + PCE**
Growth contributions by industry	(Percentage points)		
Agriculture, hunting and forestry; fishing and operation of fish hatcheries and fish farms	0.0191	0.0177	0.0368
Industry, including energy	0.9185	-0.3046	0.6139
Construction	-0.2136	0.0675	-0.1461
Wholesale and retail trade, repair of motor vehicles and household goods, hotels and restaurants; transport and communication	0.6013	-0.3535	0.2478
Financial, real estate, renting and business activities	0.1756	0.1738	0.3493
Other service activities (by category)	-0.0186	0.3992	0.3806
Sum	**1.4822**	**0.0000**	**1.4822**
Value-added growth			**1.4822**

Source: Author's calculations based on procedures and data as explained in Table 6 above. This Table 7 is the same as Table 2, p. 17 in Dumagan (2018a).

Tables 1 to 7 demonstrate the worldwide validity of the claim that—if the weights used in ALP and GDP growth decompositions are the appropriate weights of the underlying GDP quantity index, namely, chained Fisher, direct Laspeyres, or chained Laspeyres in current practice—there are no residual price change effects in ALP growth and GDP growth.

Conclusion

In the TWD framework, Diewert (2015) claimed that ALP growth could change due to changes in industry relative prices even in the special case where all industry labor productivities and all labor shares remain constant. However, Diewert (2016) found it empirically puzzling that price change effects did not matter much in the aggregate and then explained this puzzle by a Taylor series approximation showing that the price effects sum to zero to the accuracy of a first order, using shares of GDP in current prices as the weight of industry growth contributions.

In contrast, this paper showed that the effects of price changes on industry growth contributions to ALP growth will sum to zero exactly so that ALP growth will not change if the weight of an industry's contribution is appropriate, which is the weight of the industry's contribution to the growth of the GDP quantity index. The chained Fisher, direct Laspeyres, and chained Laspeyres underlie GDP in current practice worldwide. The appropriate weights are the Fisher weight—from the Fisher additive decomposition—for the chained Fisher (e.g., United States); share of GDP in constant prices for direct Laspeyres (e.g., Philippines); and share of GDP in current prices for chained Laspeyres (e.g., Italy).

In the general case where industry labor productivities, labor shares, and relative prices change, this paper finds that price effects will still sum to zero exactly if the appropriate weights are used for industry growth contributions. Therefore, in TWD, economy-wide ALP growth depends only on changes in labor productivities and in labor shares, which is the same finding in TRAD decomposition where there is no role for industry relative prices in ALP growth.

This paper's finding that changes in relative prices, even when labor productivities and labor shares change, have no effect on ALP growth is consistent with the notion in traditional theory that growth is pure quantity growth with no residual price effects. This result is due to the fact that in TWD, relative prices perform only a "homogenizing" valuation function to make GDP additive across industries. This finding of no residual price effects in ALP growth applies to GDP growth as well because the latter growth is a special case of the former when labor employment in each industry remains the same (i.e., labor shares also remain the same).

Finally, from their comments on earlier versions of this paper, the overall finding of "zero aggregate effect of relative price changes on ALP or GDP growth" in TWD appears to have gained the concession by Diewert but not by Tang and Wang. To be fair, Tang and Wang only asserted the non-zero aggregate price effect on ALP or GDP growth when the GDP index is chained Fisher. However, this paper's analytic and empirical findings from an illustration of the Fisher additive decomposition with the U.S. GDP data based on chained Fisher—showing that the aggregate price effect on GDP growth is zero—disprove Tang and Wang's assertion.

In principle, this paper argued analytically that the "appropriate" weights of industry contributions to ALP growth or GDP growth are the industry weights in the quantity index underlying each country's GDP, which is either chained Fisher, direct Laspeyres, or chained Laspeyres in current practice. With these weights, this paper showed empirically that price change effects in industry contributions to ALP growth or GDP growth would sum to zero or wash out so that economy-wide growth is purely quantity growth in any country worldwide.

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Appendix Table 1
Growth of GDP and Aggregate Labour Productivity (ALP) in the United States, 2009-2010

	GDP in current prices (billions of dollars)		GDP in chained prices (billions of 2009 dollars)		GDP Growth Contributions (percentage points)		GDP Growth
					PGE	PCE	PGE + PCE
	2009	2010	2009	2010	2010	2010	2010
Farms	109.8	129.7	109.8	111.5	0.0118	0.1154	0.1272
Forestry, fishing, and related activities	27.9	30.5	27.9	28.8	0.0062	0.0092	0.0155
Oil and gas extraction	184.6	209.3	184.6	158.6	-0.1803	0.3341	0.1538
Mining, except oil and gas	65.8	77.3	65.8	70.1	0.0298	0.0435	0.0733
Support activities for mining	40.0	45.1	40.0	46.0	0.0416	-0.0100	0.0316
Utilities	250.8	267.0	250.8	274.4	0.1637	-0.0737	0.0900
Construction	577.3	541.6	577.3	551.6	-0.1782	-0.1147	-0.2930
Wood products	20.7	22.1	20.7	21.4	0.0049	0.0030	0.0079
Nonmetallic mineral products	37.3	36.2	37.3	37.3	0.0000	-0.0107	-0.0107
Primary metals	40.1	48.4	40.1	38.4	-0.0118	0.0653	0.0535
Fabricated metal products	117.9	120.3	117.9	129.2	0.0784	-0.0718	0.0066
Machinery	115.6	122.1	115.6	127.8	0.0846	-0.0498	0.0349
Computer and electronic products	228.9	249.0	228.9	255.8	0.1866	-0.0680	0.1186
Electrical equipment, appliances, and components	50.2	50.0	50.2	51.3	0.0076	-0.0132	-0.0056
Motor vehicles, bodies and trailers, and parts	48.4	92.9	48.4	99.7	0.3558	-0.0549	0.3008
Other transportation equipment	111.9	112.2	111.9	112.3	0.0028	-0.0101	-0.0073
Furniture and related products	23.1	22.2	23.1	23.3	0.0014	-0.0095	-0.0081
Miscellaneous manufacturing	80.2	81.2	80.2	81.9	0.0118	-0.0117	0.0001
Food and beverage and tobacco products	243.2	229.7	243.2	233.4	-0.0680	-0.0449	-0.1129
Textile mills and textile product mills	15.1	15.6	15.1	15.5	0.0028	-0.0006	0.0022
Apparel and leather and allied products	9.9	10.5	9.9	10.8	0.0062	-0.0030	0.0033
Paper products	58.5	55.3	58.5	53.4	-0.0354	0.0085	-0.0268
Printing and related support activities	39.3	38.8	39.3	39.8	0.0035	-0.0102	-0.0067
Petroleum and coal products	114.6	130.0	114.6	98.5	-0.1117	0.2076	0.0959
Chemical products	310.3	330.8	310.3	330.4	0.1394	-0.0249	0.1145
Plastics and rubber products	61.5	63.3	61.5	65.3	0.0264	-0.0192	0.0072
Wholesale trade	822.8	868.5	822.8	848.3	0.1769	0.0674	0.2442
Motor vehicle and parts dealers	133.0	148.1	133.0	143.2	0.0707	0.0216	0.0923
Food and beverage stores	133.7	136.7	133.7	142.4	0.0603	-0.0510	0.0094
General merchandise stores	131.7	132.7	131.7	120.3	-0.0791	0.0749	-0.0042
Other retail	443.7	451.4	443.7	457.1	0.0929	-0.0773	0.0156
Air transportation	63.7	72.2	63.7	70.1	0.0444	0.0085	0.0529
Rail transportation	33.7	35.0	33.7	34.4	0.0049	0.0012	0.0061
Water transportation	16.6	15.9	16.6	13.9	-0.0187	0.0125	-0.0062
Truck transportation	109.3	113.3	109.3	119.8	0.0728	-0.0546	0.0183
Transit and ground passenger transportation	27.2	28.0	27.2	27.3	0.0007	0.0025	0.0032
Pipeline transportation	14.0	18.7	14.0	17.3	0.0229	0.0081	0.0310
Other transportation and support activities	88.4	95.7	88.4	91.0	0.0180	0.0246	0.0426
Warehousing and storage	45.9	46.2	45.9	47.8	0.0132	-0.0150	-0.0018
Publishing industries, except internet (includes software)	175.8	182.4	175.8	184.6	0.0610	-0.0305	0.0305
Motion picture and sound recording industries	90.8	107.5	90.8	107.4	0.1151	-0.0083	0.1068
Broadcasting and telecommunications	372.3	370.8	372.3	372.7	0.0028	-0.0442	-0.0415
Data processing, internet publishing, and other information services	66.5	69.5	66.5	70.4	0.0270	-0.0121	0.0150
Federal Reserve banks, credit intermediation, and related activities	399.5	410.2	399.5	388.3	-0.0777	0.1175	0.0399
Securities, commodity contracts, and investments	186.7	199.5	186.7	192.2	0.0381	0.0339	0.0721
Insurance carriers and related activities	357.6	365.2	357.6	359.7	0.0146	0.0076	0.0221
Funds, trusts, and other financial vehicles	25.5	31.0	25.5	28.6	0.0215	0.0140	0.0355
Housing	1464.9	1479.0	1464.9	1480.0	0.1047	-0.1308	-0.0261
Other real estate	275.7	304.9	275.7	315.2	0.2740	-0.0970	0.1770
Rental and leasing services and lessors of intangible assets	164.2	161.8	164.2	162.4	-0.0125	-0.0177	-0.0302
Legal services	213.9	206.2	213.9	198.2	-0.1089	0.0382	-0.0707
Computer systems design and related services	180.4	189.9	180.4	193.1	0.0881	-0.0381	0.0500
Miscellaneous professional, scientific, and technical services	606.0	625.8	606.0	618.5	0.0867	-0.0018	0.0849
Management of companies and enterprises	247.0	268.2	247.0	266.0	0.1318	-0.0072	0.1246
Administrative and support services	376.4	394.0	376.4	398.4	0.1526	-0.0635	0.0891
Waste management and remediation services	37.2	45.5	37.2	44.3	0.0492	0.0045	0.0538
Educational services	163.0	169.5	163.0	164.8	0.0125	0.0184	0.0309
Ambulatory health care services	498.3	517.6	498.3	505.1	0.0472	0.0433	0.0905
Hospitals	343.6	345.9	343.6	339.1	-0.0312	0.0182	-0.0130
Nursing and residential care facilities	120.1	124.0	120.1	122.1	0.0139	0.0028	0.0167
Social assistance	89.1	91.7	89.1	89.6	0.0035	0.0069	0.0104
Performing arts, spectator sports, museums, and related activities	78.8	78.8	78.8	78.3	-0.0035	-0.0031	-0.0066
Amusements, gambling, and recreation industries	60.1	65.7	60.1	66.7	0.0458	-0.0124	0.0333
Accommodation	106.6	110.7	106.6	111.6	0.0347	-0.0155	0.0192
Food services and drinking places	276.8	285.5	276.8	284.7	0.0548	-0.0184	0.0364
Other services, except government	329.5	332.4	329.5	323.9	-0.0388	0.0311	-0.0077
National defense	373.4	396.1	373.4	385.5	0.0839	0.0403	0.1243
Nondefense	230.2	247.5	230.2	238.9	0.0603	0.0389	0.0993
Government enterprises	60.3	57.5	60.3	56.0	-0.0298	0.0056	-0.0242
General government	1304.0	1332.3	1304.0	1295.2	-0.0610	0.1457	0.0847
Government enterprises	97.8	104.4	97.8	104.3	0.0451	-0.0080	0.0370
Sum of Industry Contributions to GDP Level and Growth	**14418.6**	**14964.5**	**14418.6**	**14783.8**	**2.2651**	**0.2677**	**2.5328**
Total Labour Employment (thousands)			**131709.0**	**130716.0**			
ALP Growth				**3.31**			
PCE Contribution to ALP Growth (assuming industry labour productivities and labour shares are all constant)						**0.27**	

Source: GDP and employment data are from the US Bureau of Economic Analysis. PGE, PCE, and PCE Contribution to ALP Growth (last row) are the author's calculations based on formulas in the text according to the TWD growth decomposition where the weights are shares of GDP in current prices.

Appendix Table 2

Growth of Aggregate Fisher Quantity Index and GDP in the United States, 2009-2010

	Aggregate Fisher Quantity Index			GDP Growth Contributions		GDP Growth
	Weights	Level	Growth	PGE	PCE	PGE + PCE
			(percentage points)			
	2010	2010	2010	2010	2010	2010
Farms	0.0082	0.0083	0.0126	0.0126	0.0006	0.0132
Forestry, fishing, and related activities	0.0020	0.0020	0.0064	0.0064	0.0000	0.0064
Oil and gas extraction	0.0147	0.0126	-0.2072	-0.2072	0.0017	-0.2055
Mining, except oil and gas	0.0048	0.0051	0.0311	0.0311	0.0002	0.0313
Support activities for mining	0.0027	0.0031	0.0409	0.0409	-0.0001	0.0408
Utilities	0.0170	0.0186	0.1602	0.1602	-0.0004	0.1598
Construction	0.0394	0.0376	-0.1752	-0.1752	-0.0006	-0.1758
Wood products	0.0014	0.0015	0.0049	0.0049	0.0000	0.0049
Nonmetallic mineral products	0.0025	0.0025	0.0000	0.0000	-0.0001	-0.0001
Primary metals	0.0031	0.0030	-0.0132	-0.0132	0.0003	-0.0129
Fabricated metal products	0.0078	0.0086	0.0751	0.0751	-0.0004	0.0747
Machinery	0.0078	0.0086	0.0821	0.0821	-0.0003	0.0818
Computer and electronic products	0.0155	0.0174	0.1826	0.1826	-0.0004	0.1822
Electrical equipment, appliances, and components	0.0034	0.0035	0.0075	0.0075	-0.0001	0.0074
Motor vehicles, bodies and trailers, and parts	0.0032	0.0066	0.3409	0.3409	-0.0003	0.3406
Other transportation equipment	0.0077	0.0077	0.0028	0.0028	-0.0001	0.0027
Furniture and related products	0.0016	0.0016	0.0013	0.0013	0.0000	0.0013
Miscellaneous manufacturing	0.0055	0.0056	0.0116	0.0116	-0.0001	0.0116
Food and beverage and tobacco products	0.0166	0.0159	-0.0669	-0.0669	-0.0002	-0.0671
Textile mills and textile product mills	0.0010	0.0011	0.0028	0.0028	0.0000	0.0028
Apparel and leather and allied products	0.0007	0.0007	0.0061	0.0061	0.0000	0.0061
Paper products	0.0041	0.0037	-0.0357	-0.0357	0.0000	-0.0357
Printing and related support activities	0.0027	0.0027	0.0034	0.0034	-0.0001	0.0033
Petroleum and coal products	0.0091	0.0078	-0.1283	-0.1283	0.0010	-0.1273
Chemical products	0.0214	0.0227	0.1383	0.1383	-0.0002	0.1382
Plastics and rubber products	0.0042	0.0044	0.0257	0.0257	-0.0001	0.0256
Wholesale trade	0.0573	0.0590	0.1775	0.1775	0.0003	0.1777
Motor vehicle and parts dealers	0.0093	0.0100	0.0713	0.0713	0.0001	0.0714
Food and beverage stores	0.0090	0.0096	0.0587	0.0587	-0.0003	0.0584
General merchandise stores	0.0095	0.0087	-0.0824	-0.0824	0.0004	-0.0821
Other retail	0.0303	0.0312	0.0916	0.0916	-0.0004	0.0912
Air transportation	0.0044	0.0049	0.0447	0.0447	0.0000	0.0447
Rail transportation	0.0023	0.0024	0.0049	0.0049	0.0000	0.0049
Water transportation	0.0012	0.0010	-0.0199	-0.0199	0.0001	-0.0198
Truck transportation	0.0073	0.0080	0.0703	0.0703	-0.0003	0.0700
Transit and ground passenger transportation	0.0019	0.0019	0.0007	0.0007	0.0000	0.0007
Pipeline transportation	0.0010	0.0012	0.0236	0.0236	0.0000	0.0236
Other transportation and support activities	0.0062	0.0064	0.0183	0.0183	0.0001	0.0185
Warehousing and storage	0.0031	0.0032	0.0129	0.0129	-0.0001	0.0128
Publishing industries, except internet (includes software)	0.0120	0.0126	0.0602	0.0602	-0.0002	0.0600
Motion picture and sound recording industries	0.0062	0.0074	0.1142	0.1142	-0.0001	0.1142
Broadcasting and telecommunications	0.0255	0.0256	0.0027	0.0027	-0.0003	0.0025
Data processing, internet publishing, and other information services	0.0045	0.0048	0.0267	0.0267	-0.0001	0.0266
Federal Reserve banks, credit intermediation, and related activities	0.0282	0.0275	-0.0792	-0.0792	0.0006	-0.0786
Securities, commodity contracts, and investments	0.0131	0.0135	0.0385	0.0385	0.0002	0.0387
Insurance carriers and related activities	0.0248	0.0249	0.0146	0.0146	0.0000	0.0146
Funds, trusts, and other financial vehicles	0.0018	0.0020	0.0222	0.0222	0.0001	0.0223
Housing	0.1007	0.1018	0.1038	0.1038	-0.0008	0.1030
Other real estate	0.0187	0.0213	0.2673	0.2673	-0.0005	0.2668
Rental and leasing services and lessors of intangible assets	0.0113	0.0111	-0.0124	-0.0124	-0.0001	-0.0125
Legal services	0.0150	0.0139	-0.1102	-0.1102	0.0002	-0.1100
Computer systems design and related services	0.0123	0.0132	0.0866	0.0866	-0.0002	0.0864
Miscellaneous professional, scientific, and technical services	0.0419	0.0428	0.0865	0.0865	-0.0001	0.0864
Management of companies and enterprises	0.0171	0.0184	0.1312	0.1312	-0.0001	0.1312
Administrative and support services	0.0257	0.0273	0.1505	0.1505	-0.0004	0.1501
Waste management and remediation services	0.0026	0.0031	0.0495	0.0495	0.0000	0.0495
Educational services	0.0114	0.0115	0.0126	0.0126	0.0001	0.0126
Ambulatory health care services	0.0347	0.0352	0.0473	0.0473	0.0002	0.0475
Hospitals	0.0239	0.0236	-0.0313	-0.0313	0.0001	-0.0312
Nursing and residential care facilities	0.0083	0.0085	0.0139	0.0139	0.0000	0.0139
Social assistance	0.0062	0.0062	0.0035	0.0035	0.0000	0.0035
Performing arts, spectator sports, museums, and related activities	0.0054	0.0054	-0.0035	-0.0035	0.0000	-0.0035
Amusements, gambling, and recreation industries	0.0041	0.0046	0.0451	0.0451	-0.0001	0.0450
Accommodation	0.0073	0.0076	0.0343	0.0343	-0.0001	0.0342
Food services and drinking places	0.0191	0.0196	0.0544	0.0544	-0.0001	0.0543
Other services, except government	0.0230	0.0226	-0.0390	-0.0390	0.0001	-0.0389
National defense	0.0260	0.0269	0.0844	0.0844	0.0002	0.0845
Nondefense	0.0161	0.0167	0.0609	0.0609	0.0002	0.0611
Government enterprises	0.0042	0.0039	-0.0300	-0.0300	0.0000	-0.0299
General government	0.0910	0.0904	-0.0614	-0.0614	0.0006	-0.0608
Government enterprises	0.0067	0.0072	0.0447	0.0447	0.0000	0.0447
Sum of Industry Contributions	**1.0000**	**1.0217**	**2.1705**	**2.1705**	**0.0000**	**2.1705**

Source: Author's calculations based on the same GDP data in Appendix Table 1 and formulas in the text where the industry shares of GDP in current prices used by TWD are replaced by Fisher weights above from the Fisher additive decomposition.