Modifying the "Generalized Exactly Additive Decomposition" of GDP and Aggregate Labor Productivity Growth in Practice for Consistency with Theory

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Modifying the "Generalized Exactly Additive Decomposition" of GDP and Aggregate Labor Productivity Growth in Practice for Consistency with Theory

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Abstract

The generalized exactly additive decomposition (GEAD) of GDP and aggregate labor productivity (ALP) growth, originated by Tang and Wang (2004), is gaining attention in the literature and acceptance in practice. This paper shows, however, that the original GEAD is not always consistent with the "theory" that aggregate GDP growth is *pure* quantity growth and ALP growth depends *only* on productivity and labor share changes. This paper modifies the original GEAD for consistency, subject to certain requirement, depending on the GDP quantity index that in current practice is either (1) chained Laspeyres, (2) direct Laspeyres, or (3) chained Fisher. GEAD employs *relative price* to obtain contributions that exactly add up to GDP or ALP growth. Sector contributions equal *pure growth effect* plus *price change effect* (PCE) to GDP growth and *with-in sector productivity growth effect* plus *inter-sectoral reallocation effect* to ALP growth. When relative prices change, a sector's PCE could be positive, zero, or negative but this paper shows that consistency with the above theory requires the Sum of PCE = 0 for all sectors. That is, there are no residual price effects. However, the original GEAD yields Sum of PCE = 0 only if the GDP quantity index is chained Laspeyres and, therefore, this paper modifies GEAD for theoretical consistency if the index is direct Laspeyres or chained Fisher. The findings are globally relevant because these three indexes underpin GDP in all countries in current practice.

Keywords: Growth decomposition; productivity analysis; relative prices; index number theory

JEL classification: C43, O47

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The growth of GDP is an indicator of overall economic well-being while the growth of aggregate labor productivity (ALP) is a significant determinant of growth of GDP per capita that is widely recognized as a measure of the standard of living. Hence, determining sources of growth (i.e., growth decomposition) in a manner consistent with theory is important in practice for well-grounded and informed growth policies to promote economic welfare.

The "traditional" growth decomposition (Denison, 1962, 1967) is still the method of choice in countries with GDP in constant prices. However, when countries started adopting GDP in chained prices in the mid-1990s, a need arose for a "new" decomposition because the traditional method worked exactly for GDP in constant prices by virtue of its additivity that is not a property of GDP in chained prices. Tang and Wang (2004) introduced a new decomposition employing "relative price." Relative price, which is the ratio of a sector's GDP deflator to the economy's GDP deflator, acts as the weight of a sector's GDP to obtain a weighted sum equal to GDP either in constant or in chained prices (i.e., general) and enables decompositions of growth of GDP or ALP into contributions that exactly add up to aggregate growth. Thus, Dumagan (2013) called Tang and Wang's original framework as the generalized exactly additive decomposition (GEAD).

By introducing relative price, GEAD added price effects to quantity effects in growth contributions. In both GEAD and traditional decompositions, the quantity effect in GDP (or ALP) growth comes from the growth of a sector's GDP (or labor productivity). In GEAD, the price effect comes from the change in the sector's relative price and appears separately in GDP growth but is combined with the effect of a change in labor share in ALP growth. In contrast, by ignoring relative prices, traditional decomposition does not have the price effects in GDP or ALP growth.

This paper argues that growth decomposition should be consistent with the theory that aggregate growth is pure quantity growth with no residual price effects and growth contributions are based on the specific index formula underlying the GDP under analysis to ensure accuracy. Therefore, this paper proposes that the GEAD quantity effect should be the sector's contribution to growth of the GDP quantity index that in current practice is either (1) chained Laspeyres, for example, in Italy and EU countries (European Union, 2007; Schreyer, 2004); (2) direct Laspeyres (Balk, 2010), for example, in the Philippines and many countries; or (3) chained Fisher, for example, in Canada and the U.S. (see Chevalier, 2003 for Canada; Landefeld & Parker, 1997 for the U.S.). Thus, the quantity effect must be the same in GEAD and in traditional decompositions.

Price effects in the original GEAD come from changes in relative prices, the ratios of each sector's GDP deflator to the overall GDP deflator. In theory, these deflators could be based on the same price index formula where the overall deflator is the weighted sum of the sector deflators and the weights sum to one. This theoretical relationship implies that price effects must sum to zero across all sectors so that GDP growth equals the sum of quantity effects only (unless as shown later the index is not "consistent-in-aggregation"). Accordingly, ALP growth depends only on the sectoral growth of labor productivities and changes in labor shares.[1] Hence, violating the condition that price effects must sum to zero implies that sector contributions to GDP and ALP growth are inaccurate and could be misleading. Unfortunately, the above condition is not always satisfied by Tang and Wang's original GEAD, and this finding is a major motivation for this paper.

[1] This is similar to the implication of Equation (4) in Dumagan and Balk (2016) that relative price effects in ALP vanish in the aggregate if the deflators of sectoral and aggregate nominal value-added are Sato-Vartia price indexes. Expression (26) in this paper embodies the same result although the price indexes are not Sato-Vartia.

It may be noted that Tang and Wang (2004) applied GEAD originally to ALP growth. However, as shown later, ALP growth yields GDP growth as a special case when labor employment remains the same in each sector. For this reason, this paper attributes GEAD decomposition of GDP growth also to Tang and Wang. These growth decompositions use value-added or GDP as the economy's output measure. However, for a sector, Tang & Wang (2004) analyzed value-added as a *value* in itself but in 2014, Tang and Wang analyzed value-added as a *formula*, the difference between gross output and intermediate inputs. In 2004, Tang and Wang employed a single relative price given by the ratio of a sector's value-added deflator to the aggregate GDP deflator. However, in 2014, they employed two relative prices, which are the ratios of a sector's gross output deflator and intermediate input deflator to the aggregate GDP deflator. This paper follows the analytic treatment of value-added in Tang and Wang's (2004) framework with the above single relative price, and it is this framework where the condition that Sum of PCE = 0 specifically applies.

The next section of this paper presents Tang and Wang's (2004) original GEAD decompositions of growth of GDP and ALP. To introduce the terminology, a sector's contribution to GDP growth equals pure growth effect (PGE) from the growth of GDP plus price change effect (PCE) from the change of relative price. The sector's corresponding contribution to ALP growth equals with-in sector productivity growth effect (WSPGE) from the growth of labor productivity plus inter-sectoral reallocation effect (ISRE) from the changes in relative price and labor share. Hence, in GEAD, GDP growth = Sum of PGE + Sum of PCE; ALP growth = Sum of WSPGE + Sum of ISRE.

As noted earlier, the definition of relative price in Tang and Wang (2004) implies Sum of PCE = 0 so that GDP growth = Sum of PGE. That is, while PCE could be positive, zero, or negative for a sector, GDP growth in GEAD is pure quantity growth in the aggregate, as in traditional decomposition. Moreover, Sum of PCE $= 0$ implies that although ISRE of a sector includes effects of changes in labor shares and relative prices, the Sum of ISRE includes only the effects of changes in labor shares. In GEAD, therefore, ALP growth $=$ Sum of WSPGE $+$ Sum of ISRE where ALP growth depends only on the growth of labor productivities and changes in labor shares, which hold in traditional decomposition.

However, that Sum of PCE $= 0$ in Tang and Wang's (2004) original GEAD holds true only if the GDP quantity index is *chained Laspeyres* (e.g., in Italy noted above). Therefore, the succeeding section modifies the above GEAD to satisfy Sum of PCE $= 0$ for theoretical consistency in cases where the GDP quantity index is direct Laspeyres (e.g., in the Philippines) or chained Fisher (e.g., in Canada and the U.S.). The modifications rectify in principle the applications of the original GEAD to Canada and the U.S. by Tang and Wang (2004, 2014) and to the Philippines and the U.S., except the application to Italy, by Dumagan (2013, 2014a, 2014b, 2016, 2017). The analytic results are illustrated empirically with actual GDP and employment data.

GEAD Decomposition of GDP and ALP Growth

GEAD growth decomposition begins with the aggregation of GDP levels of sectors or industries from published national accounts in period t, for example, quarter or year, which provide data on nominal GDP, Y_t, and Y_t^j, and real GDP, X_t, and X_t^j. Y_t and X_t represent the economy's GDP while Y_t^j and X_t^j represent the GDP of a sector or industry denoted by j. Henceforth, sector and industry will be used interchangeably in this paper.

GEAD Aggregation of Industry GDP Levels

By definition, X_t is obtained by dividing Y_t by the economy's GDP price index or deflator, $P_{0,t}$, so that X_t is valued in prices of the base year denoted by 0. Industry real GDP, X_t^j, is similarly obtained from industry nominal GDP, Y_t^j, using the industry's GDP deflator, $P_{0,t}^j$. From these definitions, the deflators $P_{0,t}$ and $P_{0,t}^j$ may be obtained *implicitly* by

$$P_{0,t} \equiv \frac{Y_t}{X_t} \quad ; \quad P_{0,t}^j \equiv \frac{Y_t^j}{X_t^j} \quad ; \quad Y_t = \sum_j Y_t^j . \tag{1}$$

Nominal GDP is additive while real GDP may not be additive. If $P_{0,t}$ and $P_{0,t}^j$ are direct Paasche price indexes, real GDP is in constant prices that is additive or $X_t = \sum_j X_t^j$. However, if these deflators are chained Paasche or Fisher price indexes, real GDP is in chained prices that is not additive (Balk, 2010) or $X_t \neq \sum_j X_t^j$, except in the base year. However, (1) implies that:

$$P_{0,t}\, X_t = \sum_j P_{0,t}^j X_t^j \quad ; \quad r_t^j \equiv \frac{P_{0,t}^j}{P_{0,t}} \quad ; \quad X_t = \sum_j r_t^j X_t^j . \tag{2}$$

The last part in Equation (2) is Tang and Wang's (2004) GEAD level aggregation of industry GDP that applies to GDP in constant or in chained prices, that is, regardless of the index formulas underlying the deflators, $P_{0,t}$ and $P_{0,t}^j$ (Dumagan, 2013). In Equation (2), $r_t^j \equiv P_{0,t}^j / P_{0,t}$ is relative price, the ratio of an industry's GDP deflator to the economy's GDP deflator. Thus, r_t^j is a real price using aggregate GDP as the numeraire good that converts industry real GDP, X_t^j, to $r_t^j X_t^j$ that is now in the same (i.e., homogeneous) unit as the economy's real GDP. Therefore, $r_t^j X_t^j$ is additive across industries even if X_t^j is not additive.

GEAD Decomposition of GDP Growth

Since Equation (2) holds for all *t*, the relative change in real GDP is

$$\frac{X_t}{X_{t-1}} = \sum_j r_t^j \, w_{t-1}^j \frac{X_t^j}{X_{t-1}^j} \quad ; \quad w_{t-1}^j \equiv \frac{X_{t-1}^j}{X_{t-1}} . \tag{3}$$

Note that Equations (1), (2), and (3) yield[2]

$$r_{t-1}^j w_{t-1}^j = \left(\frac{Y_{t-1}^j / X_{t-1}^j}{Y_{t-1} / X_{t-1}}\right)\left(\frac{X_{t-1}^j}{X_{t-1}}\right) = \frac{Y_{t-1}^j}{Y_{t-1}} \quad ; \quad \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} = 1 . \tag{4}$$

From Equations (3) and (4), the GEAD decomposition of growth of GDP in chained or in constant prices is

$$\text{GDP growth} \equiv \frac{X_t}{X_{t-1}} - 1 = \sum_j \left[\frac{Y_{t-1}^j}{Y_{t-1}}\left(\frac{X_t^j}{X_{t-1}^j} - 1\right) + \frac{X_t^j}{X_{t-1}}\left(r_t^j - r_{t-1}^j\right)\right] ; \tag{5}$$

$$= \sum_j \frac{Y_{t-1}^j}{Y_{t-1}}\left(\frac{X_t^j}{X_{t-1}^j} - 1\right) + \sum_j \left(\frac{r_t^j X_t^j}{X_{t-1}} - \frac{Y_{t-1}^j}{Y_{t-1}} \frac{X_t^j}{X_{t-1}^j}\right) . \tag{6}$$

The GEAD decomposition in Equation (6) consists of two parts:[3]

$$\text{PGE (pure growth effect)} \equiv \frac{Y_{t-1}^j}{Y_{t-1}}\left(\frac{X_t^j}{X_{t-1}^j} - 1\right) ; \tag{7}$$

$$\text{PCE (price change effect)} \equiv \frac{X_t^j}{X_{t-1}}\left(r_t^j - r_{t-1}^j\right) = \frac{r_t^j X_t^j}{X_{t-1}} - \frac{Y_{t-1}^j}{Y_{t-1}} \frac{X_t^j}{X_{t-1}^j} ; \tag{8}$$

$$\text{GDP growth} = \text{Sum of PGE} + \text{Sum of PCE} . \tag{9}$$

The values of PGE in Equation (7) and PCE in Equation (8) could be positive, zero, or negative from which the contribution of an industry to GDP growth is the sum PGE + PCE in real terms. PGE is real because it is due to the growth of X_{t-1}^j to X_t^j that are deflated values and, hence, real. PCE is also real because it is due to a change of relative price r_{t-1}^j to r_t^j that use GDP as the

[2] In Equation (3), $\sum_j w_{t-1}^j$ is not necessarily equal to 1 except when the deflators in (1) are direct Paasche price indexes so that X_{t-1}^j and X_{t-1} are in constant prices and, therefore, additive or $X_{t-1} = \sum_j X_{t-1}^j$ and $\sum_j w_{t-1}^j = 1$.

[3] PGE above is the same as PGE in Dumagan (2014a, 2014b) while PCE in Dumagan (2014a, 2014b) is PCE = GPIE + RPE where the latter terms stand for GPIE (growth-price interaction effect) and RPE (relative price effect). Moreover, PGE and PCE above are the same as those in Dumagan (2016, 2017).

numeraire. A positive (negative) PCE implies that a unit of an industry's output, X_t^j, will now fetch more (less) "GDP baskets" than before in exchange. Thus, being real changes, both PGE and PCE should be counted as part of an industry's growth contribution. However, theory requires that GDP growth wholly represents only "quantity" change. That is, PCE matters only for individual industries but not in the aggregate. As shown later, this implies that in Equation (9), Sum of PCE = 0 so that GDP growth = Sum of PGE.

GEAD Decomposition of ALP Growth

To produce industry GDP, X_t^j, labor employment is L_t^j. Total employment is $L_t = \sum_j L_t^j$ to produce aggregate GDP, X_t. To formalize the relationship between ALP and industry labor productivity, let Z_t be ALP and Z_t^j be industry labor productivity. Hence,

$$Z_t \equiv \frac{X_t}{L_t} \quad ; \quad Z_t^j \equiv \frac{X_t^j}{L_t^j} \quad ; \quad L_t = \sum_j L_t^j \quad ; \quad l_t^j \equiv \frac{L_t^j}{L_t} \quad ; \quad \sum_j l_t^j = 1 \,. \tag{10}$$

Combining Equation (10) with Equations (1) to (3) yields:

$$Z_t = \sum_j \frac{P_{0,t}^j}{P_{0,t}} \frac{L_t^j}{L_t} \frac{X_t^j}{L_t^j} = \sum_j r_t^j \, l_t^j \, Z_t^j \quad ; \quad Z_{t-1} = \sum_j r_{t-1}^j \, l_{t-1}^j \, Z_{t-1}^j \,. \tag{11}$$

Let G_t be the growth rate of Z_t and G_t^j the growth rate of Z_t^j. That is,

$$G_t = \frac{Z_t - Z_{t-1}}{Z_{t-1}} \quad ; \quad G_t^j = \frac{Z_t^j - Z_{t-1}^j}{Z_{t-1}^j} \,. \tag{12}$$

Substituting Equation (11) into Equation (12) yields Tang and Wang's (2004) original GEAD expression of ALP growth as[4]

$$G_t = \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} G_t^j + \sum_j \frac{Z_{t-1}^j}{Z_{t-1}} \left(r_t^j l_t^j - r_{t-1}^j l_{t-1}^j\right) G_t^j + \sum_j \frac{Z_{t-1}^j}{Z_{t-1}} \left(r_t^j l_t^j - r_{t-1}^j l_{t-1}^j\right) . \tag{13}$$

[4] For alternative labor productivity growth decompositions involving relative prices, see Diewert (2010, 2015) and Dumagan and Balk (2016).

Dumagan (2013) derived Equation (13) in detail and defined an industry's contribution to ALP growth as consisting of [5]

$$\text{WSPGE (within-sector productivity growth effect)} \equiv \frac{Y_{t-1}^{j}}{Y_{t-1}} G_t^j\ ; \qquad (14)$$

$$\text{DSRE (dynamic structural reallocation effect)} \equiv \frac{Z_{t-1}^{j}}{Z_{t-1}} \left(r_t^j l_t^j - r_{t-1}^j l_{t-1}^j\right) G_t^j\ ; \qquad (15)$$

$$\text{SSRE (static structural reallocation effect)} \equiv \frac{Z_{t-1}^{j}}{Z_{t-1}} \left(r_t^j l_t^j - r_{t-1}^j l_{t-1}^j\right). \qquad (16)$$

WSPGE is the industry's labor productivity growth weighted by its share in nominal GDP. DSRE and SSRE are reallocation effects due to changes in relative prices and labor shares. DSRE is related to the Baumol (1967) effect that indicates Baumol's "growth disease" when resources are absorbed by "stagnant" industries, those with low values of $\left(Z_{t-1}^j / Z_{t-1}\right) G_t^j$, but have high values of $\left(r_t^j l_t^j - r_{t-1}^j l_{t-1}^j\right)$ from increasing labor shares, given relative prices. SSRE is related to the Denison (1967) effect on ALP growth from changes in industry labor shares, given relative prices.[6]

For the purposes of this paper, add DSRE to SSRE above to obtain

$$\text{ISRE (inter-sector reallocation effect)} \equiv \text{DSRE} + \text{SSRE} = \frac{Z_t^j}{Z_{t-1}} \left(r_t^j l_t^j - r_{t-1}^j l_{t-1}^j\right). \quad (17)$$

Using Equation (17), ALP growth in Equation (13) becomes

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[5] Except for differences in notation, Equation (13) is identical to Tang and Wang's (2004, p. 426) equation (5).

[6] Dumagan (2013) adopted the terms WSPGE, DSRE, and SSRE from the ALP growth decomposition by Usui (2011) where, in contrast, relative prices are absent. In the terminology by Nordhaus (2002), WSPGE corresponds to pure productivity growth effect; DSRE to the Baumol (1967) effect; and SSRE to the Denison (1967) effect. However, these two effects pre-dated Tang and Wang's (2004) introduction of relative prices and, thus, equal DSRE and SSRE when $r_{t-1}^j = r_t^j = 1$.

$$\text{ALP growth} \equiv G_t = \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} G_t^j + \sum_j \frac{Z_t^j}{Z_{t-1}} \left(r_t^j l_t^j - r_{t-1}^j l_{t-1}^j \right) \; ; \tag{18}$$

$$= \text{Sum of WSPGE} + \text{Sum of ISRE} \, . \tag{19}$$

To see the connection between GDP growth and ALP growth, use Equations (1) to (4) and (10) to (17) to rewrite Equation (18) as:

$$\frac{X_t/X_{t-1}}{L_t/L_{t-1}} - 1 = \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} \left(\frac{X_t^j/X_{t-1}^j}{L_t^j/L_{t-1}^j} - 1 \right) + \sum_j \left(\frac{r_t^j X_t^j/X_{t-1}}{L_t/L_{t-1}} - \frac{Y_{t-1}^j}{Y_{t-1}} \frac{X_t^j/X_{t-1}^j}{L_t^j/L_{t-1}^j} \right) \; ; \tag{20}$$

$$\text{WSPGE} \equiv \frac{Y_{t-1}^j}{Y_{t-1}} \left(\frac{X_t^j/X_{t-1}^j}{L_t^j/L_{t-1}^j} - 1 \right) \; ; \tag{21}$$

$$\text{ISRE} \equiv \frac{r_t^j X_t^j/X_{t-1}}{L_t/L_{t-1}} - \frac{Y_{t-1}^j}{Y_{t-1}} \frac{X_t^j/X_{t-1}^j}{L_t^j/L_{t-1}^j} \, . \tag{22}$$

Thus, in GEAD, ALP growth in Equation (20) yields GDP growth in Equation (6) as a special case when labor employment stays the same in each industry (i.e., $L_{t-1}^j = L_t^j$ so that $L_{t-1} = L_t$).

Aggregate Relative Price Effects: Sum of PCE in GDP and Sum of ISRE in ALP

While Tang and Wang (2004) deserved credit for introducing relative price effects—defined by PCE in GDP growth and ISRE in ALP growth—they did not examine how their aggregate effects are related to the underlying GDP quantity index. This relationship is examined below.

Note that X_t/X_{t-1} is the economy's implicit GDP quantity index and X_t^j/X_{t-1}^j is the implicit GDP quantity index of an industry. Depending on the index formula underlying GDP, consistency-in-aggregation (CIA) means that X_t/X_{t-1} equals the weighted sum of X_t^j/X_{t-1}^j where the weights sum to 1 (Balk, 1996; Diewert, 1978; Vartia, 1976). Therefore, if the GDP quantity index is CIA, Equations (2) to (8) yield

$$\text{Sum of PCE} = \sum\nolimits_j \left(\frac{r_t^j X_t^j}{X_{t-1}} - \frac{Y_{t-1}^j}{Y_{t-1}} \frac{X_t^j}{X_{t-1}^j} \right) = \frac{X_t}{X_{t-1}} - \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}} \frac{X_t^j}{X_{t-1}^j} = 0\,; \qquad (23)$$

$$\frac{X_t}{X_{t-1}} - 1 = \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right) \quad ; \quad \text{GDP growth} = \text{Sum of PGE}\,. \qquad (24)$$

While PCE in Equation (8) could be positive, zero, or negative for an individual industry, Equation (23) shows that the Sum of PCE $= 0$ if the GDP quantity index is CIA. As a result, Equation (24) shows that PGE is the same as the industry's contribution to the growth of the GDP quantity index.

The above results have important implications for ALP growth decomposition. Note that no labor reallocation means that the labor shares are constant. That is, $L_{t-1}^j / L_{t-1} = L_t^j / L_t$ so that $L_t / L_{t-1} = L_t^j / L_{t-1}^j$. Hence, from Equations (22) and (23),

$$\text{Sum of ISRE} = \frac{L_{t-1}}{L_t} \left(\frac{X_t}{X_{t-1}} - \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}} \frac{X_t^j}{X_{t-1}^j} \right) = \frac{L_{t-1}}{L_t} \times \text{ Sum of PCE}$$

$$= 0\,. \qquad (25)$$

That is, while CIA implies that the Sum of PCE $= 0$ in GDP growth, CIA and constant labor shares imply that the Sum of ISRE $= 0$ in ALP growth.

In Equation (22), ISRE of an individual industry could be positive, zero, or negative depending on the combined effects of changes in both relative prices and labor shares. However, given the CIA with changing labor shares, Equation (22) yields

$$\text{Sum of ISRE} = \frac{X_t / X_{t-1}}{L_t / L_{t-1}} - \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}} \frac{X_t^j / X_{t-1}^j}{L_t^j / L_{t-1}^j} \neq 0\,. \qquad (26)$$

Notice that relative price, r_t^j, does not appear in Equation (26). This means that, given CIA, relative price effects cancel out in the aggregate so that the Sum of ISRE depends only on changes in labor shares.

The CIA property of the GDP quantity index suffices for the Sum of PCE $= 0$ and, hence, makes PGE and PCE consistent with the theory that, in the aggregate, GDP growth represents only quantity effects or that price effects completely cancel out. Similarly, this property makes WSPGE and ISRE consistent with the same theory that, in the aggregate, ALP growth must represent only quantity effects noted above and labor reallocation effects (see footnote 1).

In principle, Sum of PCE $= 0$ is necessary for the theoretical consistency of PGE, PCE, WSPGE, and ISRE with or without the CIA property. That Sum of PCE $= 0$ comes from the definition of relative price, $r_t^j \equiv P_{0,t}^j / P_{0,t}$ in Equation (2), and holds given the CIA property. However, without this property as in the case of the Fisher index, Sum of PCE $= 0$ is possible in GEAD growth decomposition but it requires the same level of data detail used to compute the aggregate Fisher index in the first place.

At the lowest level of detail (i.e., commodity level), the numerator of r_t^j measures relative price change, p_t^i / p_0^i, from 0 to t for each commodity i and the denominator, $P_{0,t}$, is the aggregate price index computed as the weighted sum of all price relatives where the weights sum to one. Thus, as the commodities are grouped by industry, each industry price index, $P_{0,t}^j$, is computed as the weighted sum of the price relatives in the industry group according to the same formula for the aggregate price index, $P_{0,t}$. If the aggregate quantity index underlying GDP is CIA, the corresponding aggregate price index is also CIA, for example, the pair of Laspeyres quantity and Paasche price indexes.[7] In this case, the given value of $P_{0,t}$ can be computed as the weighted sum of $P_{0,t}^j$, where the weights sum to one, no matter the level of aggregation defined

[7] If nominal GDP is Y_0 in period 0 and Y_t in period t, Fisher (1922) showed that $Y_t / Y_0 = P_{0,t}^P \times Q_{0,t}^L = P_{0,t}^F \times Q_{0,t}^F$ where $P_{0,t}^P$ is a Paasche price index; $Q_{0,t}^L$ is a Laspeyres quantity index; $P_{0,t}^F$ is a Fisher price index; and $Q_{0,t}^F$ is a Fisher quantity index. $P_{0,t}^P$ and $Q_{0,t}^L$ are CIA but not $P_{0,t}^F$ and $Q_{0,t}^F$ (Balk, 1996; Diewert, 1978; Vartia, 1976).

by j. Therefore, while $r_t^j \equiv P_{0,t}^j / P_{0,t}$ could be less than, equal to, or greater than 1 for an individual industry at any t, the aggregate relative price is always 1 under CIA because the weighted sum of $P_{0,t}^j$ equals $P_{0,t}$. This implies that PCE could be positive, zero, or negative for an industry but Sum of PCE $= 0$ no matter the level of j because the aggregate relative price, as noted above, is always 1. Consequently, CIA simplifies the computations of PGE and PCE in GDP growth and of WSPGE and ISRE in ALP growth decompositions. For example, as illustrated later in this paper, CIA implies that the value of PGE for an industry group is the same when PGE is computed by treating the group as one whole or as the sum of the individual PGEs of group members. This applies as well to PCE.

An aggregate index may be expressed as the weighted sum of the sub-indexes. CIA of the aggregate index requires that weights that sum to 1 exist at any level of the sub-indexes. The Fisher index does not satisfy this requirement and, thus, is not CIA although it has an "additive" decomposition (Balk, 2004; Dumagan, 2002; Van IJzeren, 1952) where weights that sum to 1 exist at the lowest level, when relative price change is p_t^i / p_0^i or relative quantity change q_t^i / q_0^i from 0 to t for each commodity i. However, lack of CIA means that if Fisher sub-indexes are constructed from p_t^i / p_0^i or q_t^i / q_0^i, weights that sum to 1 may not exist to make the weighted sum of the above sub-indexes equal to the aggregate Fisher index. However, while CIA is sufficient for Sum of PCE $= 0$ in Tang and Wang's GEAD as in the case of the Laspeyres index, CIA is not necessary. It will be shown that GEAD may be modified to yield Sum of PCE $= 0$ when GDP is in chained prices based on Fisher, although this index is not CIA.

It follows from above that Tang and Wang's GEAD formulas for GDP growth in Equation (6) and ALP growth in Equation (20) are not necessarily uniformly applicable to GDP based on chained Laspeyres, direct Laspeyres, or chained Fisher, which are the three quantity indexes underlying GDP in current practice. It is shown later that Equations (6) and (20) are perfectly suitable (i.e., without modification) only to GDP based on chained Laspeyres. Hence, they need to be modified when GDP is based on direct Laspeyres or chained Fisher. With the appropriate modification, the Sum of PCE $= 0$ so that all contributions to the growth of GDP and ALP comply with the theory that they are free of residual price effects.

Applying GEAD in Practice

In current practice, the quantity indexes underpinning GDP are chained Laspeyres, direct Laspeyres, or chained Fisher.

GDP Based on the Chained Laspeyres Quantity Index

Let Y_0 and Y_0^j be the economy's and an industry's GDP in the base year 0. With the same base year, let $Q_{0,t-1}$ and $Q_{0,t}$ be the economy's GDP quantity indexes in years $t-1$ and t with $Q_{0,t-1}^j$ and $Q_{0,t}^j$ as the industry GDP quantity indexes. Hence, by definition of real GDP[8]

$$X_{t-1} \equiv Y_0 Q_{0,t-1} \quad ; \quad X_t \equiv Y_0 Q_{0,t} \quad ; \quad X_{t-1}^j \equiv Y_0^j Q_{0,t-1}^j \quad ; \quad X_t^j \equiv Y_0^j Q_{0,t}^j \,. \tag{27}$$

Combining (23) and (27) yields

$$\text{Sum of PCE} \equiv \frac{Q_{0,t}}{Q_{0,t-1}} - \sum\nolimits_j \frac{Y_{t-1}^j}{Y_{t-1}} \left(\frac{Q_{0,t}^j}{Q_{0,t-1}^j} \right). \tag{28}$$

In Equation (28), industry and aggregate nominal GDP are obtained from prices, p_{it}^j, and quantities, q_{it}^j, of $i = 1, 2, \cdots, N$ commodities (i.e., goods and services) produced by $j = 1, 2, \cdots, M$ industries. By earlier definitions, these prices and quantities yield

[8] Real GDP may be computed either by deflating nominal GDP by a price index or inflating base-year nominal GDP by a quantity index. For example, real GDP for the economy is $X_t \equiv Y_t / P_{0,t} = Y_0 Q_{0,t}$ and is $X_t^j \equiv Y_t^j / P_{0,t}^j = Y_0^j Q_{0,t}^j$ for an industry. These results follow from the decomposition (Fisher, 1922) of Y_t / Y_0 into a product of pairs quantity and price indexes defined in footnote 7.

$$Y_{t-1}^{j} \equiv \sum_{i} p_{it-1}^{j} q_{it-1}^{j} \quad ; \quad Y_{t-1} \equiv \sum_{j} Y_{t-1}^{j} \quad ; \quad Y_{t}^{j} \equiv \sum_{i} p_{it}^{j} q_{it}^{j} \quad ; \quad Y_{t} \equiv \sum_{j} Y_{t}^{j} . \quad (29)$$

The expressions in Equation (29) will be handy in determining the sign of Equation (28) depending on the formula of the GDP quantity index.

In the case of chained indexes (Balk, 2010), the GDP quantity index is defined by

$$Q_{0,t-1} \equiv Q_{0,1} \times Q_{1,2} \times \cdots \times Q_{t-2,t-1} \quad ; \quad Q_{0,t} = Q_{0,t-1} \times Q_{t-1,t} . \quad (30)$$

The definition in Equation (30) applies analogously to industry chained quantity indexes. Therefore, for GDP in chained prices based on (chained) Laspeyres quantity indexes, Equations (27) to (30) yield

$$\frac{Q_{0,t}^{j}}{Q_{0,t-1}^{j}} = Q_{t-1,t}^{j} \equiv \frac{\sum_i p_{it-1}^{j} q_{it}^{j}}{\sum_i p_{it-1}^{j} q_{it-1}^{j}} \quad ; \quad \frac{Q_{0,t}}{Q_{0,t-1}} = Q_{t-1,t} \equiv \frac{\sum_j \sum_i p_{it-1}^{j} q_{it}^{j}}{\sum_j \sum_i p_{it-1}^{j} q_{it-1}^{j}} \; ; \quad (31)$$

$$Q_{t-1,t} = \sum_{j} \left(\frac{\sum_i p_{it-1}^{j} q_{it-1}^{j}}{\sum_j \sum_i p_{it-1}^{j} q_{it-1}^{j}} \right) Q_{t-1,t}^{j} \quad ; \quad \sum_{j} \left(\frac{\sum_i p_{it-1}^{j} q_{it-1}^{j}}{\sum_j \sum_i p_{it-1}^{j} q_{it-1}^{j}} \right) = 1 . \quad (32)$$

Together, Equations (28) to (32) yield[9]

$$\text{Sum of PCE} \equiv \frac{\sum_j \sum_i p_{it-1}^{j} q_{it}^{j}}{\sum_j \sum_i p_{it-1}^{j} q_{it-1}^{j}} - \sum_{j} \left(\frac{\sum_i p_{it-1}^{j} q_{it-1}^{j}}{\sum_j \sum_i p_{it-1}^{j} q_{it-1}^{j}} \right) \left(\frac{\sum_i p_{it-1}^{j} q_{it}^{j}}{\sum_i p_{it-1}^{j} q_{it-1}^{j}} \right) = 0 . \quad (33)$$

The result in Equation (33) implies that Tang and Wang's original GEAD formulas for PGE in Equation (7), PCE in Equation (8), WSPGE in Equation (21), and ISRE in Equation (22) are perfectly suitable (i.e., need no modification) for GDP (or value-added) and employment in Italy

[9] The result in (32) is the consistency-in-aggregation property (Balk, 1996; Diewert, 1978; Vartia, 1976) of the Laspeyres quantity index.

(Table 1) where value-added, like those in other EU countries, is based on the chained Laspeyres quantity index.[10]

Note in Table 1 that because of non-additivity of GDP in chained prices (Balk, 2010), aggregate GDP is not equal to the simple sum of the GDP of individual industries. This is shown by the residual (2,503 in 2009). However, this residual is immaterial in the GEAD framework because relative price weights applied to GDP of industries make them sum up to equal aggregate GDP (1,076,071 in 2009).

Table 1
Value-Added and Employment in Italy, 2009-2010

	Value-added in in current prices (millions of euros)		**Value-added in chained prices** (millions of 2000 euros)		**Employment in persons** (thousands)	
	2009	**2010**	**2009**	**2010**	**2009**	**2010**
Agriculture, hunting and forestry; fishing and operation of fish hatcheries and fish farms	25,886	26,370	28,379	28,665	967	983
Industry, including energy	260,237	268,437	208,201	218,251	4,970	4,787
Construction	84,819	82,761	55,949	54,023	1,935	1,907
Wholesale and retail trade, repair of motor vehicles and household goods, hotels and restaurants; transport and communication	304,350	307,514	253,973	260,836	6,057	6,024
Financial, real estate, renting and business activities	389,123	393,613	293,776	295,588	3,694	3,716
Other service activities (as one whole)	303,267	308,248	233,164	232,968	7,217	7,241
Public administration and defence; compulsory social security	93,644	94,962	68,573	68,281	1,342	1,333
Education	67,371	66,656	54,477	54,532	1,583	1,560
Health and social work	83,409	86,481	67,080	67,304	1,650	1,667
Other community, social and personal service activities	43,518	44,508	31,350	31,266	1,123	1,128
Private households with employed persons	15,325	15,640	11,811	11,745	1,520	1,554
Value-added and employment	**1,367,681**	**1,386,942**	**1,076,071**	**1,092,021**	**24,839**	**24,658**
Residual	**0.0**	**0.0**	**2,503**	**1,531**	**0.0**	**0.0**

Source: Istat - Istituto Nazionale di Statistica at https://www.istat.it.

Table 2 shows that the Sum of PGE = 1.4822, the actual GDP growth. CIA is confirmed by the result that the Sum of PCE = 0. Moreover, CIA is illustrated by the results in rows 6 and 7. Row 6 shows the contribution of "Other services activities (as one whole)" when the five individual service categories (Table 1) are treated as one whole group. In comparison,

[10] Brueton (1999) noted that the EU System of National Accounts 1995 recommended—as the basis for chained volume measures—Laspeyres quantity and Paasche price indexes because they are more practical than the theoretically superior Fisher quantity and price indexes adopted by Canada and the U.S. as recommended by the UN System of National Accounts 1993.

row 7 shows the sum of the contributions of these five individual service categories. CIA implies that the contribution of the group taken as one whole equals the sum of the individual contributions of the members of the group. Moreover, this equality applies to the PGE and PCE components. As shown, the PGE contribution of −0.0186 percentage points and PCE contribution of 0.3992 percentage points of the group as one whole (row 6) equals the corresponding sums of the individual PGE and PCE contributions of the group members (row 7). Hence, CIA simplifies computations in that if only group contributions are of interest, there is no need to know the individual contribution of the group members.

Table 2
Value-Added Growth in Italy, 2009-2010

	GEAD growth contributions by industry	**PGE**	**PCE** (Percentage points)	**PGE + PCE**
1	Agriculture, hunting and forestry; fishing and operation of fish hatcheries and fish farms	0.0191	0.0177	0.0368
2	Industry, including energy	0.9185	-0.3046	0.6139
3	Construction	-0.2136	0.0675	-0.1461
4	Wholesale and retail trade, repair of motor vehicles and household goods, hotels and restaurants; transport and communication	0.6013	-0.3535	0.2478
5	Financial, real estate, renting and business activities	0.1756	0.1738	0.3493
6	**Other service activities (as one whole)**	**-0.0186**	**0.3992**	**0.3806**
7	**Other service activities (by category)**	**-0.0186**	**0.3992**	**0.3806**
8	**Sum (1 to 5, and 6)**	**1.4822**	**0.0000**	**1.4822**
9	**Sum (1 to 5, and 7)**	**1.4822**	**0.0000**	**1.4822**
10	**Value-added growth**			**1.4822**

Source: Author's calculations of the *original* GEAD formulas for PGE in (7) and PCE in (8) from data in Table 1.

Finally, the results in Table 2 show that GDP growth = Sum of PGE and Sum of PCE = 0 for 2010 have an analytic basis and can be verified to be true in Italian GDP in other years. However, the result that Sum of PCE = 0 for the whole economy does not imply that the PCE of individual industries may be ignored. In GEAD, an industry's overall contribution to GDP growth is the sum of PGE + PCE (last column, Table 2) where PCE is the contribution from a change in the industry's relative price or real exchange value of its output relative to the output of other industries. Therefore, ignoring PCE simply because Sum of PCE = 0 would misrepresent the overall growth contribution of an industry.[11]

Table 3 presents the results of applying ALP growth decomposition to data in Table 1. Recall from Equation (19) that since the Sum of PCE = 0 (Table 2), then Sum of ISRE = 0 if there is no labor reallocation (i.e., constant labor shares). Therefore, the result that Sum of ISRE ≠ 0 in Table 3 must be due only to labor reallocation effects, although ISRE at the industry level is due to the combined effects of change in labor share and change in relative price.

Table 3
ALP Growth in Italy, 2009-2010

	GEAD growth contributions by industry	**WSPGE**	**ISRE** (Percentage points)	**WSPGE + ISRE**
1	Agriculture, hunting and forestry; fishing and operation of fish hatcheries and fish farms	-0.0120	0.0630	0.0510
2	Industry, including energy	1.6836	-0.9253	0.7583
3	Construction	-0.1275	0.0260	-0.1015
4	Wholesale and retail trade, repair of motor vehicles and household goods, hotels and restaurants; transport and communication	0.7253	-0.3121	0.4132
5	Financial, real estate, renting and business activities	0.0076	0.5535	0.5611
6	**Other service activities (as one whole)**	**-0.0939**	**0.6403**	**0.5464**
7	**Other service activities (by category)**	**-0.0018**	**0.5482**	**0.5464**
8	**Sum (1 to 5, and 6)**	**2.1831**	**0.0453**	**2.2284**
9	**Sum (1 to 5, and 7)**	**2.2752**	**-0.0468**	**2.2284**
10	**ALP growth**			**2.2284**

Source: Author's calculations of the *original* GEAD formulas for WSPGE in (21) and ISRE in (22) from data in Table 1.

Notice in Table 3 that WSPGE and ISRE change between rows 6 and 7 although their sum remains at 0.5464. In contrast, there is no corresponding change in PGE and PCE between rows 6 and 7 in Table 2. To explain this contrast, note that the aggregation of GDP does not affect GDP growth contributions in rows 6 and 7 of Table 2 because the underlying Laspeyres quantity

[11] A major reason for the shift in measuring real GDP from constant to chained prices is to correctly account for the effects of relative price changes on GDP growth by avoiding overestimation (underestimation) of the growth contributions of GDP components whose prices on average have fallen (risen) since the base period. In the US, a major motivation for the shift to chained prices was to correct for the overestimation of the growth contribution of information technology products whose prices were falling rapidly (Landefeld & Parker, 1997).

index is CIA. However, row 6 of Table 3 involves GDP aggregation and labor aggregation to obtain the contributions of "other service activities (as one whole)." In this case, the "correct" WSPGE and ISRE are those in row 7 based on the principle that growth contributions should be calculated at the more disaggregated level compared to row 6. Therefore, in this example, the correct WSPGE is -0.0018 and ISRE is 0.5482 for a total contribution by "other service activities" of 0.5464 percentage points to ALP growth in Italy.

GDP Based on the Direct Laspeyres Quantity Index

For GDP in constant prices, the industry and aggregate indexes in Equation (28) are direct Laspeyres quantity indexes (Balk, 2010) defined by:

$$Q_{0,t-1}^{j} \equiv \frac{\sum_i p_{i0}^{j} q_{it-1}^{j}}{\sum_i p_{i0}^{j} q_{i0}^{j}} \quad ; \quad Q_{0,t-1} \equiv \frac{\sum_j \sum_i p_{i0}^{j} q_{it-1}^{j}}{\sum_j \sum_i p_{i0}^{j} q_{i0}^{j}} \; ; \tag{34}$$

$$Q_{0,t}^{j} \equiv \frac{\sum_i p_{i0}^{j} q_{it}^{j}}{\sum_i p_{i0}^{j} q_{i0}^{j}} \quad ; \quad Q_{0,t} \equiv \frac{\sum_j \sum_i p_{i0}^{j} q_{it}^{j}}{\sum_j \sum_i p_{i0}^{j} q_{i0}^{j}} \, . \tag{35}$$

From the above definitions, the direct Laspeyres quantity index is also CIA because

$$Q_{0,t-1} = \sum\nolimits_j \left(\frac{\sum_i p_{i0}^{j} q_{i0}^{j}}{\sum_j \sum_i p_{i0}^{j} q_{i0}^{j}} \right) Q_{0,t-1}^{j} \quad ; \quad \sum\nolimits_j \left(\frac{\sum_i p_{i0}^{j} q_{i0}^{j}}{\sum_j \sum_i p_{i0}^{j} q_{i0}^{j}} \right) = 1 \; ; \tag{36}$$

$$Q_{0,t} = \sum\nolimits_j \left(\frac{\sum_i p_{i0}^{j} q_{i0}^{j}}{\sum_j \sum_i p_{i0}^{j} q_{i0}^{j}} \right) Q_{0,t}^{j} \, . \tag{37}$$

In this case, to conform to the CIA property and satisfy the Sum of PCE $= 0$, Equation (28) needs to be modified by replacing shares of GDP in current prices by shares of GDP in constant prices and then substituting Equations (34) to (37) to obtain

$$\text{Sum of PCE} \equiv \frac{\sum_j \sum_i p_{i0}^{j} q_{it}^{j}}{\sum_j \sum_i p_{i0}^{j} q_{it-1}^{j}} - \sum\nolimits_j \left(\frac{\sum_i p_{i0}^{j} q_{it-1}^{j}}{\sum_j \sum_i p_{i0}^{j} q_{it-1}^{j}} \right) \left(\frac{\sum_i p_{i0}^{j} q_{it}^{j}}{\sum_i p_{i0}^{j} q_{it-1}^{j}} \right) = 0 \, . \tag{38}$$

Hence, using * to distinguish the modified GEAD formulas for GDP in constant prices, Tang and Wang's original PGE in Equation (7), PCE in Equation (8), and Sum of PCE in Equation (23) become[12]

$$\text{PGE* (pure growth effect)} \equiv \frac{X_{t-1}^{j}}{X_{t-1}}\left(\frac{X_{t}^{j}}{X_{t-1}^{j}} - 1\right); \tag{39}$$

$$\text{PCE* (price change effect)} \equiv \frac{r_t^j X_t^j}{X_{t-1}} - \frac{X_{t-1}^{j}}{X_{t-1}}\frac{X_t^j}{X_{t-1}^j} = \frac{r_t^j X_t^j}{X_{t-1}} - \frac{X_t^j}{X_{t-1}}; \tag{40}$$

$$\text{Sum of PCE*} \equiv \sum\nolimits_j \left(\frac{r_t^j X_t^j}{X_{t-1}} - \frac{X_t^j}{X_{t-1}}\right) = \frac{X_t}{X_{t-1}} - \frac{X_t}{X_{t-1}} = 0\,. \tag{41}$$

Similarly, WSPGE in Equation (21), ISRE in Equation (22), and Sum of ISRE in Equation (26) become[13]

$$\text{WSPGE*} \equiv \frac{X_{t-1}^{j}}{X_{t-1}}\left(\frac{X_t^j/X_{t-1}^j}{L_t^j/L_{t-1}^j} - 1\right); \tag{42}$$

$$\text{ISRE*} \equiv \frac{r_t^j X_t^j/X_{t-1}}{L_t/L_{t-1}} - \frac{X_{t-1}^j}{X_{t-1}}\frac{X_t^j/X_{t-1}^j}{L_t^j/L_{t-1}^j} = \frac{r_t^j X_t^j/X_{t-1}}{L_t/L_{t-1}} - \frac{X_t^j/X_{t-1}}{L_t^j/L_{t-1}^j}; \tag{43}$$

$$\text{Sum of ISRE*} = \frac{X_t/X_{t-1}}{L_t/L_{t-1}} - \sum\nolimits_j \frac{X_t^j/X_{t-1}}{L_t^j/L_{t-1}^j}\,. \tag{44}$$

The results in Equations (41) and (44) follow from the fact that in GEAD, $X_t = \sum_j r_t^j X_t^j$ is true for any real GDP. Moreover, $X_t = \sum_j X_t^j$ because GDP in constant prices is additive.

Additivity (i.e., without the GEAD relative price weights) of GDP in constant prices simplifies the traditional (TRAD) decomposition of GDP growth into

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[12] Since for an industry, share of GDP in constant prices could differ from share of GDP in current prices only in size but not in sign, PGE* and PCE* could also differ from PGE and PCE only in size but not in sign. However, when applied to GDP in constant prices but using shares of GDP in current prices, Sum of PCE could be different from zero but, by using shares of GDP in constant prices, Sum of PCE* $= 0$ as shown above.

[13] The relationships of PGE*, PCE*, and Sum of PCE* to PCE, PGE, and Sum of PCE described in footnote 12 apply in similar fashion to the relationships of WSPGE*, ISRE*, and Sum of ISRE* to WSPGE, ISRE, and Sum of ISRE.

$$X_t = \sum_j X_t^j \quad ; \quad X_{t-1} = \sum_j X_{t-1}^j \quad ; \quad \frac{X_t}{X_{t-1}} - 1 = \sum_j \frac{X_{t-1}^j}{X_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right). \qquad (45)$$

In Equation (45), the contribution of an industry to the growth of GDP in constant prices may be defined as

$$\text{TRAD} \equiv \frac{X_{t-1}^j}{X_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right). \qquad (46)$$

The above formulas are applied to GDP and employment in the Philippines (Table 4) where GDP is in constant prices and the results are presented in Table 5.

Table 4
Gross Domestic Product and Employment in the Philippines, 2009-2010

	GDP in current prices (millions of pesos)		GDP in constant prices (millions of 2000 pesos)		Employed persons (thousands)	
	2009	**2010**	**2009**	**2010**	**2009**	**2010**
1 Agriculture, forestry, and fishery	1,049,874	1,108,718	663,744	662,665	12043	11956
2 Mining and quarrying	106,396	128,727	59,130	65,898	166	199
3 Manufacturing	1,706,391	1,930,779	1,137,534	1,264,523	2,894	3,033
4 Construction	460,426	551,230	284,994	325,820	1,891	2,017
5 Electricity, gas and water supply	271,892	321,543	184,943	203,274	142	150
6 Services (as one whole)	4,431,165	4,962,483	2,966,895	3,179,358	17,925	18,682
Transport Communication and Storage	561,093	586,197	423,398	427,766	2,679	2,723
Trade	1,359,500	1,563,786	875,616	948,743	6,736	7,034
Finance	544,526	622,404	340,329	374,716	369	400
Other Services	1,966,045	2,190,096	1,327,552	1,428,133	8,141	8,525
Gross domestic product	**8,026,143**	**9,003,480**	**5,297,240**	**5,701,539**	**35,060**	**36,037**
Residual	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**

Source: Economic and Social Database, Philippine Institute for Development Studies at www.pids.gov.ph.

Comparing Equations (39) and (46) shows TRAD and modified GEAD yield the same PGE* (Table 5). However, TRAD ignores relative prices and, therefore, yields zero PCE* for each industry while modified GEAD yields non-zero PCE* but the Sum of $\text{PCE}^* = 0$. As a result, TRAD and modified GEAD yield contributions that sum up to the same growth of GDP in constant prices.

The CIA property of the direct Laspeyres quantity index is illustrated in rows 6 and 7 (Table 5) where the contribution of Services as one whole equals the sum of the individual contributions of the members of the group. Moreover, this equality applies to PGE* and PCE*.

As shown, the PGE* contribution of 4.0108 percentage points and PCE* contribution of −0.6951 percentage points of the group as one whole (row 6) equals the corresponding sums of individual PGE* and PCE* contributions of the group members (row 7).

Table 5
GDP Growth in the Philippines, 2009-2010

		PGE*		PCE*		PGE* + PCE*	
		TRAD	**GEAD**	**TRAD**	**GEAD**	**TRAD**	**GEAD**
	Growth contributions by industry			(Percentage points)			
1	Agriculture, forestry, and fishery	-0.0204	-0.0204	0.0000	0.7446	-0.0204	0.7242
2	Mining and quarrying	0.1278	0.1278	0.0000	0.2949	0.1278	0.4226
3	Manufacturing	2.3973	2.3973	0.0000	-0.7898	2.3973	1.6074
4	Construction	0.7707	0.7707	0.0000	0.4389	0.7707	1.2096
5	Electricity, gas and water supply	0.3461	0.3461	0.0000	0.0065	0.3461	0.3526
6	**Services (as one whole)**	**4.0108**	**4.0108**	0.0000	-0.6951	**4.0108**	**3.3158**
7	**Services (by category)**	**4.0108**	**4.0108**	0.0000	**-0.6951**	**4.0108**	**3.3158**
	Total contributions to GDP growth (1 to 5, and 6)	**7.6323**	**7.6323**	**0.0000**	**0.0000**	**7.6323**	**7.6323**
	Total contributions to GDP growth (1 to 5, and 7)	**7.6323**	**7.6323**	**0.0000**	**0.0000**	**7.6323**	**7.6323**
	GDP Growth					**7.6323**	**7.6323**

Source: Author's calculations of the *modified* GEAD formulas for PGE* in (39) and PCE* in (40) from data in Table 4 which yield the same PGE* in TRAD and GEAD. TRAD yields zero PCE* by ignoring relative prices, which is equivalent to assuming that relative price equals 1 for each industry in (40).

It is important to note that relative price effects could lead to sign reversals in an industry's contribution between TRAD and (modified) GEAD. This is shown by Agriculture, fishery, and forestry where the total contribution, PGE* + PCE*, is negative in TRAD, −0.0204, but positive in GEAD, 0.7242.

Moreover, it may be noted that the procedures and results in Table 5 rectify those in Dumagan (2014a, 2014b, 2016, 2017).[14]

For ALP growth decomposition with GDP in constant prices, shares of GDP in current prices in the original GEAD formulas are replaced by shares of GDP in constant prices to obtain WSPGE* in Equation (42) and ISRE* in Equation (43). WSPGE* in modified GEAD is the

[14] Table 5 above and Table 6 in Dumagan (2017, p. 18) use the same Philippine data (Table 4 above). However, the results are different because Table 5 employs PGE* in Equation (39) and PCE* in Equation (40) for the direct Laspeyres quantity index that underlies the above GDP while Table 6 employs PGE in Equation (7) and PCE in Equation (8) that are appropriate only for GDP in chained prices based on (chained) Laspeyres. Therefore, Table 5 is a correction to Dumagan's (2017) Table 6 and to similar tables in Dumagan (2014a, 2014b, 2016).

same as in TRAD. However, ISRE* in TRAD is obtained by setting r_t^j to unity in ISRE* in Equation (43) since TRAD ignores relative prices.

Applying the above modified GEAD formulas and the corresponding TRAD formulas to the data in Table 4, the results are presented in Table 6. These results also rectify the original GEAD procedures and results in Dumagan (2017) that use the same data in Table 4.

In Table 6, ALP growth $=$ Sum of WSPGE* $+$ Sum of ISRE* where Sum of ISRE* $\neq 0$. For each industry, TRAD and GEAD yield the same WSPGE* but different ISRE* because ISRE* depends on changes in both labor shares and relative prices in GEAD while it depends only on changes in labor shares in TRAD. But overall, Sum of ISRE* is the same because price effects cancel out so that TRAD and GEAD yield the same ALP growth.

Table 6
ALP Growth in the Philippines, 2009-2010

		WSPGE*		**ISRE***		**WSPGE* + ISRE***	
		TRAD	**GEAD**	**TRAD**	**GEAD**	**TRAD**	**GEAD**
	Growth contributions by industry			(Percentage points)			
1	Agriculture, forestry, and fishery	0.0704	0.0704	-0.4298	0.2945	-0.3594	0.3649
2	Mining and quarrying	-0.0785	-0.0785	0.1726	0.4594	0.0941	0.3809
3	Manufacturing	1.2993	1.2993	0.4509	-0.3175	1.7503	0.9818
4	Construction	0.3865	0.3865	0.2175	0.6446	0.6040	1.0310
5	Electricity, gas and water supply	0.1478	0.1478	0.0943	0.1006	0.2420	0.2484
6	**Services (as one whole)**	**1.5780**	**1.5780**	**0.8060**	**0.1298**	**2.3841**	**1.7078**
7	**Services (by category)**	**1.3621**	**1.3621**	**1.0220**	**0.3458**	**2.3841**	**1.7078**
	Total contributions to ALP growth (1 to 5, and 6)	**3.4035**	**3.4035**	**1.3115**	**1.3115**	**4.7150**	**4.7150**
	Total contributions to ALP growth (1 to 5, and 7)	**3.1875**	**3.1875**	**1.5275**	**1.5275**	**4.7150**	**4.7150**
	ALP Growth					**4.7150**	**4.7150**

Source: Author's calculations of the *modified* GEAD formulas for WSPGE* in (42) and ISRE* in (43) applied to data in Table 4. TRAD is calculated assuming relative prices equal 1 in ISRE* for each industry.

Table 6 shows that relative price changes could lead to sign reversals between TRAD and GEAD, for example, in ISRE* of Agriculture, forestry, and fishery (row 1) and of Manufacturing (row 3). In turn, this could lead to sign reversal in the industry's total contribution to ALP growth, WSPGE* + ISRE*, for example, in the case of Agriculture, fishery, and forestry, which is negative in TRAD, −0.3594, but positive in GEAD, 0.3649. Thus, relative price changes may not be ignored in GDP and ALP growth decompositions.

GDP Based on the Chained Fisher Quantity Index

Tang and Wang's (2004) original GEAD also needs modification to satisfy the Sum of PCE = 0 for theoretical consistency, if applied to GDP based on chained Fisher. This finding appears ironic given that Tang and Wang first applied GEAD to Canada and the U.S. where chained Fisher underpins GDP.

By definition, a chained Fisher quantity index from the base period 0 to period t is

$$Q_{0,t-1}^{F} = Q_{0,1}^{F} \times Q_{1,2}^{F} \times \cdots \times Q_{t-2,t-1}^{F} \quad ; \quad Q_{0,t}^{F} = Q_{0,t-1}^{F} \times Q_{t-1,t}^{F} \, . \qquad (47)$$

Chained indexes for industry GDP, denoted by j, are similarly defined. Given base year nominal GDP, (Y_0, Y_0^j), real GDP in chained prices, (X_{t-1}, X_{t-1}^j), and (X_t, X_t^j), are obtained by

$$X_{t-1} = Y_0 Q_{0,t-1}^{F} \quad ; \quad X_t = Y_0 Q_{0,t}^{F} \quad ; \quad X_{t-1}^{j} = Y_0^{j} Q_{0,t-1}^{Fj} \quad ; \quad X_t^{j} = Y_0^{j} Q_{0,t}^{Fj} \, ; \qquad (48)$$

$$\frac{X_t}{X_{t-1}} = \frac{Q_{0,t}^{F}}{Q_{0,t-1}^{F}} = Q_{t-1,t}^{F} \quad ; \quad \frac{X_t^{j}}{X_{t-1}^{j}} = \frac{Q_{0,t}^{Fj}}{Q_{0,t-1}^{Fj}} = Q_{t-1,t}^{Fj} \, . \qquad (49)$$

Substituting Equation (49) into Tang and Wang's GDP growth Equation (6) yields

$$\text{GDP growth} = Q_{t-1,t}^{F} - 1 = \sum_j \frac{Y_{t-1}^{j}}{Y_{t-1}} \left(Q_{t-1,t}^{Fj} - 1 \right) + \sum_j \left(\frac{r_t^{j} X_t^{j}}{X_{t-1}} - \frac{Y_{t-1}^{j}}{Y_{t-1}} Q_{t-1,t}^{Fj} \right) . \qquad (50)$$

Let PGE^F be the pure growth effect and PCE^F be the price change effect in Equation (50). That is,

$$\text{PGE}^F = \frac{Y_{t-1}^{j}}{Y_{t-1}} \left(Q_{t-1,t}^{Fj} - 1 \right) \quad ; \quad \text{PCE}^F = \frac{r_t^{j} X_t^{j}}{X_{t-1}} - \frac{Y_{t-1}^{j}}{Y_{t-1}} Q_{t-1,t}^{Fj} \, . \qquad (51)$$

Since in GEAD, $\sum_j r_t^j X_t^j = X_t$ for any real GDP, it follows that $\sum_j r_t^j X_t^j / X_{t-1} = Q_{t-1,t}^F$. In Equation (51), PCE^F could be positive, zero, or negative but because the Fisher quantity index is not CIA, it follows that they do not totally cancel out in the aggregate. Therefore, if applied to GDP in chained prices based on the Fisher index, the original GEAD will yield

$$\text{Sum of PCE}^F = \sum_j \left(\frac{r_t^j X_t^j}{X_{t-1}} - \frac{Y_{t-1}^j}{Y_{t-1}} Q_{t-1,t}^{Fj} \right) = Q_{t-1,t}^F - \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} Q_{t-1,t}^{Fj} \neq 0 \, ; \qquad (52)$$

$$\text{GDP growth} = Q_{t-1,t}^F - 1 \neq \text{ Sum of PGE}^F = \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} \left(Q_{t-1,t}^{Fj} - 1 \right) \, . \qquad (53)$$

The original GEAD formulas in Equations (50) to (53) are applied to the U.S. GDP and employment (Table 7) where GDP is in chained prices based on Fisher and the results are shown in Table 8. The decomposition of the U.S. GDP growth in Equation (50) is exactly additive but Equations (51) to (53) imply that PGE^F and PCE^F are incorrect. Specifically, PGE^F is not equal to the industry's contribution to the growth of the Fisher quantity index and, therefore, not a pure quantity growth contribution.

The lack of CIA of the Fisher index is illustrated by the differences between the results in row 15 and row 16 (Table 8). Row 15 shows the contribution of "Finance, insurance, real estate, rental, and leasing (as one whole)" when the six financial service industries (Table 7) are treated as one whole group while row 16 shows the sum of the individual contributions of these six industries. Notice that the 〖"PGE" 〗 ^"F" contribution of "0.3565" of the group (row 15) differs from the sum of the 〖"PGE" 〗 ^"F" contributions of "0.3600" of the group members (row 16). Similarly, the 〖"PCE" 〗 ^"F" contribution of -"0.0662" of the group differs from the sum of the 〖"PCE" 〗 ^"F" contributions of -"0.0696" of the group members. However, while PGE^{F} and PCE^{F} change with regrouping, their sum remains the same, equal to 0.2903.

Table 7
Gross Domestic Product and Employment in the United States, 2009-2010

		GDP in current prices (billions of dollars)		GDP in chained prices (billions of 2009 dollars)		Employment (thousands)	
		2009	**2010**	**2009**	**2010**	**2009**	**2010**
1	Agriculture, forestry, fishing, and hunting	137.7	160.2	137.7	140.3	1907	1935
2	Mining	290.3	331.7	290.3	272.7	648	662
3	Utilities	250.8	267.0	250.8	274.4	556	546
4	Construction	577.3	541.6	577.3	551.6	7,657	7,189
5	Manufacturing	1,726.7	1,830.6	1,726.7	1,818.2	11,849	11,538
6	Wholesale trade	822.8	868.5	822.8	848.3	5,581	5,476
7	Retail trade	842.1	868.8	842.1	862.1	13,500	13,358
8	Transportation and warehousing	398.8	425.1	398.8	421.4	4,414	4,338
9	Information	705.3	730.2	705.3	735.1	2,775	2,658
10	Professional and business services	1,661.1	1,729.7	1,661.1	1,718.0	17,613	17,860
11	Educational services, health care, and social assistance	1,214.0	1,248.5	1,214.0	1,220.5	18,590	18,880
12	Arts, entertainment, recreation, accommodation, and food services	522.3	540.7	522.3	541.3	11,181	11,150
13	Other services, except government	329.5	332.4	329.5	323.9	6,820	6,684
14	Government	2,065.8	2,137.9	2,065.8	2,079.8	20,506	20,441
15	Finance, insurance, real estate, rental, and leasing (as one whole)	2,874.1	2,951.6	2,874.1	2,925.4	8,112	8,001
	Federal Reserve banks, credit intermediation, and related activities	399.5	410.2	399.5	388.3	2,562	2,507
	Securities, commodity contracts, and investments	186.7	199.5	186.7	192.2	893	869
	Insurance carriers and related activities	357.6	365.2	357.6	359.7	2,300	2,313
	Funds, trusts, and other financial vehicles	25.5	31.0	25.5	28.6	82	82
	Real estate	1,740.6	1,783.9	1,740.6	1,794.8	1,733	1,726
	Rental and leasing services and lessors of intangible assets	164.2	161.8	164.2	162.4	542	504
	Gross domestic product	**14,418.6**	**14,964.5**	**14,418.6**	**14,783.8**	**131,709**	**130,716**
	Residual	**0.0**	**0.0**	**0.0**	**38.7**	**0.0**	**0.0**

Source: Bureau of Economic Analysis.

Table 8
GDP Growth in the United States, 2009-2010

	GEAD growth contributions by industry	PGE (Percentage points)	PCE (Percentage points)	PGE + PCE (Percentage points)
1	Agriculture, forestry, fishing, and hunting	0.0180	0.1246	0.1426
2	Mining	-0.1221	0.3814	0.2594
3	Utilities	0.1637	-0.0737	0.0900
4	Construction	-0.1782	-0.1147	-0.2930
5	Manufacturing	0.6346	-0.0673	0.5673
6	Wholesale trade	0.1769	0.0674	0.2442
7	Retail trade	0.1387	-0.0263	0.1124
8	Transportation and warehousing	0.1567	-0.0099	0.1468
9	Information	0.2067	-0.0951	0.1115
10	Professional and business services	0.3946	-0.0637	0.3309
11	Educational services, health care, and social assistance	0.0451	0.0896	0.1347
12	Arts, entertainment, recreation, accommodation, and food services	0.1318	-0.0494	0.0823
13	Other services, except government	-0.0388	0.0311	-0.0077
14	Government	0.0971	0.2239	0.3210
15	**Finance, insurance, real estate, rental, and leasing (as one whole)**	**0.3565**	**-0.0662**	**0.2903**
16	**Finance, insurance, real estate, rental, and leasing (by category)**	**0.3600**	**-0.0696**	**0.2903**
17	**Sum (1 to 14, and 15)**	**2.1812**	**0.3516**	**2.5328**
18	**Sum (1 to 14, and 16)**	**2.1847**	**0.3482**	**2.5328**
19	**GDP growth**			**2.5328**

Source: Author's calculations of the *original* GEAD formulas for PGE and PCE given the Fisher index in (51) from data in Table 7.

Overall, the lack of CIA is indicated in Table 8 by the non-zero result that the Sum of $\text{PCE}^F = 0.3482$.[15] Hence, the Sum of $\text{PGE}^F = 2.1847$ is not equal to GDP growth $= 2.5328$. Since the effects of changes in relative prices do not cancel out, the Sum of PGE^F above is not pure quantity growth.

Moreover, using the implicit Fisher quantity indexes in Equation (49) from the U.S. GDP data, the original Tang and Wang formulas in Equations (21) and (22) for ALP growth contributions become

$$\text{WSPGE}^F = \frac{Y_{t-1}^j}{Y_{t-1}}\left(\frac{Q_{t-1,t}^{Fj}}{L_t^j/L_{t-1}^j} - 1\right) \quad ; \quad \text{ISRE}^F = \frac{r_t^j X_t^j/X_{t-1}}{L_t/L_{t-1}} - \frac{Y_{t-1}^j}{Y_{t-1}}\frac{Q_{t-1,t}^{Fj}}{L_t^j/L_{t-1}^j}\,. \qquad (54)$$

Table 9 presents the results of applying Equation (54) to the data in Table 7. By implication of Sum of $\text{PCE}^F = 0.3482 \neq 0$ in Table 8, Sum of $\text{ISRE}^F = 0.0809$ in Table 9 must be due to changes in both labor shares and relative prices, which is contrary to the theory.

Results similar to those in Table 9 that are inconsistent with the theory may be found in Tang and Wang (2004). First note that for each industry, WSPGE^F is equal to Tang and Wang's (2004, p. 426) pure productivity growth effect, the first term in their equation (5). In this equation, their second term is the relative size change effect and the third is the interaction term. The sum of the latter two terms for the same industry equals ISRE^F and an example is shown by the sum of column (4) and column (5) in Tang and Wang's (2004, p. 430) Table 1 for Canada and US. In light of the argument above that ISRE^F and WSPGE^F (Table 9) are objectionable for

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[15] In Table 8, Sum of $\text{PCE}^F = 0.3482$ in 2009-2010 if the industries in the sub-group (row 16) are treated individually. However, Dumagan (2017) found in the U.S. GDP data during 1997–2015 that the above sum was negative for some years. Therefore, the null hypothesis that Sum of $\text{PCE}^F = 0$ is testable. The test results showed the Sum of PCE^F was significantly different from zero in the U.S. GDP during the above period (Dumagan, 2017, p. 16). This finding only bolsters the contention that PCE^F and, by extension, PGE^F in Equation (51) are analytically incorrect.

inconsistency with theory, it follows that Tang and Wang's (2004) results in their Tables 1, 3, 5, 7, and 9—as well as their similar (2014) results—are all objectionable for the same reason.[16]

Table 9
ALP Growth in the United States, 2009-2010

	GEAD growth contributions by industry	WSPGE	ISRE	WSPGE + ISRE
		(Percentage points)		
1	Agriculture, forestry, fishing, and hunting	0.0040	0.1470	0.1510
2	Mining	-0.1621	0.4387	0.2766
3	Utilities	0.1985	-0.0946	0.1039
4	Construction	0.0708	-0.3356	-0.2648
5	Manufacturing	0.9745	-0.3119	0.6626
6	Wholesale trade	0.2897	-0.0002	0.2894
7	Retail trade	0.2023	-0.0446	0.1576
8	Transportation and warehousing	0.2079	-0.0390	0.1689
9	Information	0.4311	-0.2815	0.1495
10	Professional and business services	0.2298	0.1911	0.4209
11	Educational services, health care, and social assistance	-0.0849	0.2846	0.1997
12	Arts, entertainment, recreation, accommodation, and food services	0.1422	-0.0317	0.1105
13	Other services, except government	0.0069	0.0027	0.0096
14	Government	0.1430	0.2893	0.4323
15	**Finance, insurance, real estate, rental, and leasing (as one whole)**	**0.6380**	**-0.1940**	**0.4439**
16	**Finance, insurance, real estate, rental, and leasing (by category)**	**0.5772**	**-0.1333**	**0.4439**
17	**Sum (1 to 14, and 15)**	**3.2916**	**0.0201**	**3.3117**
18	**Sum (1 to 14, and 16)**	**3.2309**	**0.0809**	**3.3117**
19	**ALP growth**			**3.3117**

Source: Author's calculations of the *original* GEAD formulas for WSPGE and ISRE given the Fisher index in (54) from data in Table 7.

For theoretical consistency, Tang and Wang's original GEAD needs modification for Sum of $\text{PCE}^F = 0$. However, the fact that Fisher is not CIA implies that exact decomposition of growth of the U.S. GDP can only be done from the same data detail used for computing the aggregate Fisher quantity index, $Q^F_{t-1,t}$. In this case, the modification for Sum of $\text{PCE}^F = 0$ is possible by means of the additive decomposition of the Fisher index (Balk, 2004; Dumagan, 2002; Van IJzeren, 1952) that the U.S. Bureau of Economic Analysis uses to determine contributions to the U.S. GDP growth (Moulton & Seskin, 1999).[17] This decomposition is

[16] Similarly, the results for the U.S. in Dumagan (2013, 2014a, 2014b, 2016, 2017) obtained using Equation (54) are objectionable.

[17] Balk (2004) credited Dumagan (2002) with an "independent rediscovery" of the additive decomposition of Fisher that Balk noted was earlier derived by Van IJzeren (1952) but was not widely known because it was written in Dutch and published in an "obscure Dutch journal." BEA's growth contribution formula (Moulton & Seskin, 1999) is adopted from the decomposition by Van IJzeren that looks very different from the decomposition by Dumagan. However, Balk showed these decompositions are equivalent.

$$Q^F_{t-1,t} = \sqrt{Q^L_{t-1,t} \times Q^P_{t-1,t}} = \sum\nolimits_i w^F_{it-1} \left(\frac{q_{it}}{q_{it-1}}\right). \tag{55}$$

By definition, Equation (55) shows the Fisher quantity index is the geometric mean of Laspeyres, $Q^L_{t-1,t}$, and Paasche, $Q^P_{t-1,t}$, quantity indexes. The right-hand expression is the additive decomposition using the same relative growth of individual commodity quantities, q_{it}/q_{it-1}, in the geometric mean formula. The Fisher weight, w^F_{it-1}, is defined by

$$w^F_{it-1} = \left(\frac{P^F_{t-1,t}}{P^L_{t-1,t} + P^F_{t-1,t}}\right) w^L_{it-1} + \left(\frac{P^L_{t-1,t}}{P^L_{t-1,t} + P^F_{t-1,t}}\right) w^P_{it-1} \quad ; \quad \sum\nolimits_i w^F_{it-1} = 1\,. \tag{56}$$

In Equation (56), $P^F_{t-1,t}$ is Fisher price index and $P^L_{t-1,t}$ is Laspeyres price index. Moreover, w^L_{it-1} is Laspeyres weight and w^P_{it-1} is Paasche weight defined from prices, p_{it-1} and p_{it}, and quantities, q_{it-1} and q_{it}, by

$$w^L_{it-1} = \frac{p_{it-1}q_{it-1}}{\sum_i p_{it-1}q_{it-1}} \quad ; \quad w^P_{it-1} = \frac{p_{it}q_{it-1}}{\sum_i p_{it}q_{it-1}} \quad ; \quad \sum\nolimits_i w^L_{it-1} = \sum\nolimits_i w^P_{it-1} = 1\,. \tag{57}$$

Hence, using Equation (49),

$$\text{GDP growth} = \frac{X_t}{X_{t-1}} - 1 = \frac{Q^F_{0,t}}{Q^F_{0,t-1}} - 1 = Q^F_{t-1,t} - 1 = \sum\nolimits_i w^F_{it-1} \left(\frac{q_{it}}{q_{it-1}} - 1\right). \tag{58}$$

Note that starting from Equation (55), there are no more industry groupings of commodities. Therefore, to avoid confusion, the jth industry in earlier discussions may now be replaced by the ith commodity in the modified GEAD for the Fisher framework. Henceforth,

$$Y_t = \sum\nolimits_i Y^i_t \quad ; \quad Y^i_t = p_{it}q_{it} \quad ; \quad P^{Fi}_{t-1,t} = \frac{p_{it}}{p_{it-1}} \quad ; \quad r^i_t = \frac{P^{Fi}_{t-1,t}}{P^F_{t-1,t}}\,; \tag{59}$$

$$X^i_t = \frac{Y^i_t}{P^{Fi}_{t-1,t}} = p_{it-1}q_{it} \quad ; \quad X_t = \frac{Y_t}{P^F_{t-1,t}} \quad ; \quad \sum\nolimits_j \frac{r^i_t X^i_t}{X_{t-1}} = \sum\nolimits_i \frac{1}{X_{t-1}} \frac{Y^i_t}{P^F_{t-1,t}}\,. \tag{60}$$

Therefore, combining Equations (58) to (60), the modified GEAD decomposition of the growth of U.S. GDP is

$$\text{GDP growth} = \frac{X_t}{X_{t-1}} - 1 = \sum\nolimits_i w_{it-1}^F \left(\frac{q_{it}}{q_{it-1}} - 1\right)$$

$$+ \sum\nolimits_i \left(\frac{1}{X_{t-1}} \frac{Y_t^i}{P_{t-1,t}^F} - w_{it-1}^F \frac{q_{it}}{q_{it-1}}\right). \tag{62}$$

Using * to denote modified GEAD, an industry's contribution to GDP growth in Equation (62) consists of

$$\text{PGE}^{F*} = w_{it-1}^F \left(\frac{q_{it}}{q_{it-1}} - 1\right) \quad ; \quad \text{PCE}^{F*} = \frac{1}{X_{t-1}} \frac{Y_t^i}{P_{t-1,t}^F} - w_{it-1}^F \frac{q_{it}}{q_{it-1}} \ . \tag{63}$$

PGE^{F*} and PCE^{F*} are exactly additive and theoretically consistent because Equations (58) to (63) imply

$$\text{GDP growth} = \text{Sum of PGE}^{F*} = \sum\nolimits_i w_{it-1}^F \left(\frac{q_{it}}{q_{it-1}} - 1\right) ; \tag{64}$$

$$\text{Sum of PCE}^{F*} = \frac{X_t}{X_{t-1}} - \sum\nolimits_i w_{it-1}^F \frac{q_{it}}{q_{it-1}} = 0 \, . \tag{65}$$

Finally, following Equations (20) and (62), the modified GEAD decomposition of the U.S. ALP growth with GDP based on chained Fisher may be expressed as

$$\text{ALP growth} = \frac{X_t/X_{t-1}}{L_t/L_{t-1}} - 1 = \sum\nolimits_i w_{it-1}^F \left(\frac{q_{it}/q_{it-1}}{L_t^i/L_{t-1}^i} - 1\right)$$

$$+ \sum\nolimits_i \left(\frac{\left(Y_t^i/P_{t-1,t}^F\right)}{X_{t-1}(L_t/L_{t-1})} - w_{it-1}^F \frac{q_{it}/q_{it-1}}{L_t^i/L_{t-1}^i}\right) ; \tag{66}$$

$$L_{t-1} = \sum\nolimits_i L_{t-1}^i \quad ; \quad L_t = \sum\nolimits_i L_t^i \, . \tag{67}$$

In Equation (66), L_{t-1}^i and L_t^i are the labor inputs for q_{it-1} and q_{it-1} comprising total employment in Equation (67).

It is important to note that Tang and Wang's original GEAD formulas use shares of nominal GDP in the preceding year as weights and, thus, are the same as the modified GEAD formulas in Equations (62) to (66) after substituting the Laspeyres weight, w_{it-1}^{L}, for the Fisher weight, w_{it-1}^{F}. However, because these weights are unequal according to Equation (56), the substitution violates Equation (65) and, therefore, implies that Tang and Wang's original GEAD formulas are inconsistent with theory when applied to GDP in chained prices based on the Fisher index in Canada and the U.S. regardless of the level of data aggregation.

Theoretical inconsistency in the above sense means that the inequality from zero in Equation (52) holds, in which case the values of industry contributions to GDP growth from Equation (51) and ALP growth from Equation (54) are inaccurate. However, it is possible that the absolute difference from zero above diminishes when the data is more detailed. That is, at lower levels of aggregation, growth contributions could become less inaccurate. But any inaccuracy is problematic because it could mean inaccuracy in size, in sign, or in both. Unfortunately, inaccuracy in sign (i.e., sign reversal)—implying the result is misleading—is possible with changes in the level of aggregation if the GDP quantity index is not CIA, like the Fisher index. This underscores the importance of the theoretically consistent modified GEAD decompositions of GDP growth in Equation (62) and ALP growth in Equation (66) because they avoid sign reversals in industry contributions to GDP and ALP growth in the Fisher index framework of the U.S. and Canada.

Moreover, it may be noted that Reinsdorf (2015) proposed a new decomposition of ALP growth based on the Fisher index employing simplifications by approximations for practicability. However, the simplifications appear unnecessary in principle in light of this paper's modified GEAD decomposition of ALP growth in Equation (66) that is exactly consistent with the additive decomposition of the Fisher quantity index in Equation (55).

Finally, it is unfortunate that detailed commodity price, quantity, and labor employment are not obtainable from publicly available U.S. national income or product and labor data. For this reason, it is not possible to illustrate exactly additive and theoretically consistent contributions to the growth of GDP and ALP in the U.S. according to this paper's modified GEAD framework for the Fisher index.

Conclusion

This paper argues that growth decomposition should be consistent with the theory that aggregate growth is pure quantity growth with no residual price effects, and growth contributions are based on the specific index formula underlying the GDP under analysis to ensure accuracy. Against this theory, this paper re-examined Tang and Wang's (2004) GEAD decomposition of GDP and ALP growth.

In GEAD, Tang and Wang introduced relative price to obtain industry growth contributions that exactly add up to the growth of GDP or ALP given GDP in constant or in chained prices. Thus, GEAD growth contributions comprise quantity and price effects. Quantity effects in GDP growth come from the growth of industry GDP and those in ALP growth come from a combination of industry GDP growth and changes in labor shares. Price effects from changes in relative prices stand alone in GDP growth but are combined with changes in labor shares in ALP growth.

However, Tang and Wang did not examine the relationship of relative price changes to the underlying GDP quantity index and, thus, did not care about the sum of price effects, let alone this paper's argument that this sum should be zero. That is, in the view of this paper, the effects of relative price changes matter only for individual industries but not in the aggregate. This condition is necessary for aggregate GDP growth to be pure quantity growth (i.e., no residual price effects) and for ALP growth to depend only on changes in industry labor productivities and labor shares. Of the three quantity indexes underpinning GDP in all countries in current practice, namely, (1) chained Laspeyres, for example, in Italy, (2) direct Laspeyres, for example, in the Philippines, and (3) chained Fisher, for example, in the U.S., Tang and Wang's original GEAD framework satisfied the above condition only if the quantity index is in chained Laspeyres.

Therefore, this paper provided modifications to the original GEAD for theoretical consistency when the GDP quantity index is direct Laspeyres or chained Fisher. Given the direct Laspeyres, GDP is in constant prices and the modification is simply replacing the shares of GDP in current prices—in the starting period of the growth decomposition—in the original GEAD by shares of GDP in constant prices also in the above starting period. This modification is all that is needed and applies to any level of aggregation because the direct Laspeyres is CIA. In contrast, the modification given the chained Fisher consists of two necessary steps. One is replacing the above shares of GDP in current prices in the original GEAD by Fisher weights obtained from the additive decomposition of the Fisher index. Because the Fisher index is not CIA, the other necessary step is to implement the additive decomposition at the same level of data detail or aggregation used to compute the aggregate Fisher index in the first place. Otherwise, there will remain residual (i.e., non-zero) price effects.

The modifications in this paper correct for theoretical consistency—by ensuring that the sum of price effects is zero—the applications of the original GEAD by Tang and Wang (2004, 2014) to Canada and the U.S. and by Dumagan (2013, 2014a, 2014b, 2016, 2017) to the Philippines and the U.S., except the application to Italy. Finally, it may be claimed that the analytic and empirical findings of this paper are globally relevant because the three kinds of GDP analyzed cover GDP in all countries in current practice.

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