Consistent Level Aggregation and Growth Decomposition of Real GDP

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This paper formulates a general framework for consistent level aggregation and growth decomposition of real GDP. However, the focus is on US GDP in chained prices based on the Fisher index since this GDP motivated this paper's purposes. These are to explain why problematic residuals–in contributions to US GDP level and growth "not allocated by industry"–show up in the existing framework by the Bureau of Economic Analysis and, therefore, to propose an alternative framework for consistent level aggregation and growth decomposition where residuals cannot arise. This paper's residual-free framework applies to real GDP regardless of the underlying indexes, i.e., to GDP either in chained prices or in constant prices.

Key Words: Real GDP; relative prices; index numbers; aggregation; additivity.

JEL classification: C43, O47

I. Introduction

Consistent real GDP means that GDP level and growth are *invariant* to changes in groupings of the same components, e.g., changes in classifications of existing industries. Moreover, given the industries, there should be *no* residuals in industry contributions to the level and growth of real GDP either in chained prices or in constant prices.

Thus, US GDP in chained prices appears inconsistent in view of the residuals in industry contributions to level and growth computed by the Bureau of Economic Analysis (BEA), the official compiler of the US *National Income and Product Accounts*. These residuals are considered unavoidable (Ehemann, Katz, and Moulton, 2002; Whelan, 2002) because the Fisher

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index formula underlying US GDP is inconsistent in aggregation (Diewert, 1978).[1]

However, this paper finds that the *implicit* (i.e., computed value) US GDP Fisher quantity index can be expressed as an *exact* weighted sum of the *implicit* industry GDP Fisher quantity indexes. Based on this finding, this paper posits that BEA residuals are avoidable because a framework for consistent level aggregation and growth decomposition of US GDP in chained prices is possible. This framework incorporates differences and changes in relative prices that chained indexes are designed to capture but upon closer examination are surprisingly *ignored* by BEA in computing industry contributions to level and growth of US GDP. Thus, accounting for relative prices is the key in this paper's framework to eliminate the BEA residuals.

While this paper focuses on US GDP in chained prices based on the Fisher index, the analytic results apply with equal validity to GDP in chained prices based on other indexes as well as to GDP in constant prices. That is, the framework of this paper applies to *any* real GDP.

The rest of this paper is organized as follows. Section II presents US GDP with the problematic residuals and shows how these residuals arise from existing BEA procedures. Section III presents this paper's procedures for *exact* level aggregation and growth decomposition and shows that these procedures eliminate the above residuals. Section IV concludes this paper.

II. Explaining Residuals in Level and Growth of US GDP in Chained Prices

To put the problems of this paper in focus, consider US GDP in Table 1. GDP in current dollars is additive so that the zero residuals in the bottom of columns 1 and 2 imply that there are *no* missing industries in Table 1. Therefore, the level residuals (bottom of columns 3 and 4) and growth residuals (bottom of column 5) are due to BEA procedures for the Fisher index

[1] For illustration, calculate an index value in a "single" stage using all data at once. Now, separate the data into subsets and, using the same formula, calculate index values from the subsets. The "two-stage" index value is the weighted sum–where the weights sum to one–of the separate index values. In this case, the index is "consistent in aggregation" (Vartia, 1976; Balk, 1996) if the single and two-stage index values are equal, which is satisfied by the Laspeyres and Paasche indexes because they can be explicitly (i.e., as formulas) expressed as *exact* weighted sums–where the weights sum to one–of their corresponding subindexes. This exact weighted sum is not, however, possible for the explicit (i.e., formula) Fisher index and, thus, this index is inconsistent in aggregation.

framework of US GDP in chained prices (Landefeld and Parker, 1997; Seskin and Parker, 1998; Moulton and Seskin, 1999).[2]

Table 1. Level and Growth of US GDP

	BEA				
	GDP in Current Prices (billions of current dollars)		**GDP in Chained Prices** (billions of chained 2009 dollars)		**GDP growth** (percent)
	2011	2012	2011	2012	2012
US GDP Level and growth	**15,533.8**	**16,244.6**	**15,052.4**	**15,470.7**	**2.78**
Industry contributions to US GDP level and growth					
Agriculture, forestry, fishing, and hunting	197.7	201.1	134.6	135.0	0.00
Mining	409.3	429.7	301.0	343.3	0.35
Utilities	280.0	275.1	284.6	289.6	0.03
Construction	546.1	581.1	549.1	571.2	0.14
Durable goods	1,006.7	1,065.3	1,040.8	1,083.2	0.26
Nondurable goods	916.2	969.0	813.1	808.8	-0.03
Wholesale trade	909.4	962.7	862.1	884.2	0.15
Retail trade	894.6	927.8	872.0	883.5	0.08
Transportation and warehousing	447.8	471.6	437.4	442.1	0.03
Information	741.3	776.7	745.3	777.8	0.21
Finance and insurance	1,011.6	1,078.2	960.5	982.8	0.15
Real estate and rental and leasing	2,000.1	2,094.4	1,994.6	2,038.1	0.28
Professional, scientific, and technical services	1,079.1	1,140.2	1,051.1	1,094.9	0.29
Management of companies and enterprises	282.9	307.7	279.8	302.5	0.15
Administrative and waste management services	462.8	489.4	452.2	468.7	0.11
Educational services	174.0	182.3	165.0	166.6	0.01
Health care and social assistance	1,109.1	1,157.4	1,072.9	1,101.8	0.19
Arts, entertainment, and recreation	150.3	157.3	150.6	154.0	0.02
Accommodation and food services	412.5	439.2	414.8	426.4	0.07
Other services, except government	337.5	352.0	322.3	328.3	0.04
Federal government	715.1	711.7	681.5	674.4	-0.05
State and local government	1,449.8	1,474.5	1,390.9	1,394.3	0.02
Residuals: "Not allocated by industry"	**0**	**0**	**58.0**	**96.6**	**0.28**

Source: Bureau of Economic Analysis (BEA), released on April 25, 2014.

To start the analysis, consider two succeeding periods $t-1$ and t, e.g., 2011 and 2012 in Table 1, with data for GDP in current prices, Y_{t-1}, Y_{t-1}^{j}; Y_t, Y_t^{j}, and GDP in chained prices, X_{t-1}, X_{t-1}^{j}; X_t, X_t^{j}, where values with superscript j are for industries and those without j are for the US. By the "factor reversal" and "product" tests (Fisher, 1922), the relative change of Y_{t-1} to Y_t

[2] There are also residuals in GDP in current dollars but they are rounding errors that become zero when reduced to whole numbers. The residuals in chained 2009 dollars equal US GDP less the simple sum of the *most* detailed components computed by BEA. Since industry GDP in chained dollars is sensitive to level of detail, the 2012 residual of 96.6 billion, for example, is not equal to US GDP less the simple sum of industry GDP in Table 1 because the industries in this table are above the most detailed level.

equals the product of the Fisher price index linking $t-1$ to t, $P_{t-1,t}$, and the Fisher quantity index, $Q_{t-1,t}$. That is,[3]

$$(1)\quad \frac{Y_t}{Y_{t-1}} = P_{t-1,t} \times Q_{t-1,t} \quad ; \quad \frac{Y_t}{Y_0} = P_{0,t} \times Q_{0,t}\,.$$

In the second expression, $P_{0,t}$ and $Q_{0,t}$ are *chained* Fisher price and quantity indexes when the base period 0 is more than one period away from the current period t. By definition,

$$(2)\quad P_{0,t} \equiv \left[P_{0,1} \times P_{1,2} \times \cdots \times P_{t-2,t-1}\right] \times P_{t-1,t} = P_{0,t-1} \times P_{t-1,t} \quad ; \quad P_{0,0} \equiv 1\,;$$

$$(3)\quad Q_{0,t} \equiv \left[Q_{0,1} \times Q_{1,2} \times \cdots \times Q_{t-2,t-1}\right] \times Q_{t-1,t} = Q_{0,t-1} \times Q_{t-1,t} \quad ; \quad Q_{0,0} \equiv 1\,.$$

Formulas (2) and (3) apply in the same form to each industry.

GDP in chained prices for the US, X_{t-1} and X_t, and for an industry, X_{t-1}^j and X_t^j, are by definition,

$$(4)\quad X_{t-1} \equiv \frac{Y_{t-1}}{P_{0,t-1}} = Y_0 \times Q_{0,t-1} \quad ; \quad X_t \equiv \frac{Y_t}{P_{0,t}} = Y_0 \times Q_{0,t} = Y_0 \times Q_{0,t-1} \times Q_{t-1,t}\,;$$

$$(5)\quad X_{t-1}^j \equiv \frac{Y_{t-1}^j}{P_{0,t-1}^j} = Y_0^j \times Q_{0,t-1}^j \quad ; \quad X_t^j \equiv \frac{Y_t^j}{P_{0,t}^j} = Y_0^j \times Q_{0,t}^j = Y_0^j \times Q_{0,t-1}^j \times Q_{t-1,t}^j\,.$$

The following analysis employs *implicit* indexes from (4) and (5) that can be computed from published data on nominal and real GDP like those in Table 1. These indexes are given by,

$$(6)\quad P_{0,t} = \frac{Y_t}{X_t} \quad ; \quad P_{0,t}^j = \frac{Y_t^j}{X_t^j} \quad ; \quad Q_{t-1,t} = \frac{Q_{0,t}}{Q_{0,t-1}} = \frac{X_t}{X_{t-1}} \quad ; \quad Q_{t-1,t}^j = \frac{Q_{0,t}^j}{Q_{0,t-1}^j} = \frac{X_t^j}{X_{t-1}^j}\,.$$

An aggregate nominal value equals the simple sum of its components so that $Y_t = \sum_j Y_t^j$ in (6). In contrast, an aggregate real value may not equal the simple sum of its components, which is exemplified by US GDP in chained prices in Table 1 where $X_t \neq \sum_j X_t^j$. Hence, from above,

$$(7)\quad s_t^j \equiv \frac{Y_t^j}{Y_t} \quad ; \quad w_t^j \equiv \frac{X_t^j}{X_t} \quad ; \quad r_t^j \equiv \frac{P_{0,t}^j}{P_{0,t}} = \frac{s_t^j}{w_t^j} \quad ; \quad \sum_j s_t^j = 1 \quad ; \quad \sum_j w_t^j \neq 1 \;\; if \;\; r_t^j \neq 1\,.$$

In this paper, the crucial variable is the "relative price" denoted by r_t^j, the ratio of an industry chained price index, $P_{0,t}^j$, to the US chained price index, $P_{0,t}$.

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[3] Express GDP in current prices as $Y_{t-1}^j = \sum_i p_{t-1}^{ij} q_{t-1}^{ij}$; $Y_{t-1} = \sum_j Y_{t-1}^j$; $Y_t^j = \sum_i p_t^{ij} q_t^{ij}$; and $Y_t = \sum_j Y_t^j$ from prices (p) and quantities (q). Using these prices and quantities in the Fisher formula, the equations in (1) may be verified.

This paper's finding that US GDP in chained prices is consistent in aggregation follows from (1) to (7), which together yield,

$$(8)\quad X_t = \sum_j r_t^j X_t^j \quad ; \quad \frac{X_t}{X_{t-1}} = \sum_j r_t^j w_{t-1}^j \frac{X_t^j}{X_{t-1}^j} \quad ; \quad \sum_j r_t^j w_{t-1}^j \neq 1 \ \text{ if } \ r_{t-1}^j \neq r_t^j \ ;$$

$$(9)\quad \sum_j r_t^j w_{t-1}^j = \sum_j w_{t-1}^j = 1 \ \text{ only if } \ r_{t-1}^j = r_t^j = 1 \ .$$

It is clear from (8) and (9) that US GDP, X_t, equals the weighted sum of industry GDP, X_t^j, and, in turn, the US GDP quantity index, X_t/X_{t-1}, equals the weighted sum of the industry quantity indexes, X_t^j/X_{t-1}^j, where the sum of the weights may not equal 1 unless relative prices are constant. Note that X_t^j/X_{t-1}^j may be calculated at different levels of aggregation while allowing w_{t-1}^j and r_t^j to adjust to *maintain* the value of X_t/X_{t-1}. Hence, X_t/X_{t-1} is consistent in aggregation.[4] Moreover, by using relative price r_t^j as weight of X_t^j, X_t is an exact aggregation of US GDP in chained prices. Thus, the procedures in (8) ensure that residuals cannot exist.

If relative prices are constant, price indexes are all equal in which case $r_{t-1}^j = r_t^j = 1$. In this case, the weights sum to 1 as shown in (9). Hence, if relative prices change, (8) yields,

$$(10)\quad \frac{Y_t}{P_{0,t}} \neq \sum_j \frac{Y_t^j}{P_{0,t}^j} \quad ; \quad r_t^j \equiv \frac{P_{0,t}^j}{P_{0,t}} \neq 1 \quad ; \quad X_t \neq \sum_j X_t^j \ .$$

These inequalities imply that the level residuals, e.g., 96.6 billion chained 2009 dollars in 2012 in Table 1, are due to differences in relative prices that are ignored by BEA in taking the simple or unweighted sum of industry GDP in chained prices.[5] However, by applying relative price r_t^j as weight of X_t^j, (8) completely eliminates the BEA residuals in Table 1.

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[4] This result is implied by the second equation in (8) and is illustrated later in Table 4. However, this equation does not contradict Diewert (1978) because it holds for implicit (i.e., computed values) Fisher indexes. That is, the implicit Fisher index is consistent in aggregation even though the explicit Fisher index is not, as explained in footnote 1.

[5] "Non-additivity" in (10) is universal in all countries that have adopted GDP in chained prices. For country practices, see Aspden (2000) for Australia; Brueton (1999) for the UK; Chevalier (2003) for Canada; Landefeld and Parker (1997) for the US; Maruyama (2005) for Japan; Schreyer (2004) and EU (2007) for EU and OECD countries. Brueton (1999) noted that the EU System of National Accounts 1995 recommended Paasche price and Laspeyres quantity indexes as more practical than the theoretically superior Fisher price and Fisher quantity

To explain the growth residual, e.g., 0.28 percentage points in 2012 in Table 1, let $Q_{t-1,t}^{j} = q_t^j / q_{t-1}^j$ be the lowest level of detail permissible by BEA data. In this case, BEA's growth decomposition is based on an "additive decomposition" of the Fisher quantity index where the weights sum to one. This is given by,

$$(11) \quad Q_{t-1,t}^{*} = \sum_j \omega_{t-1}^j \frac{q_t^j}{q_{t-1}^j} \quad ; \quad Q_{t-1,t}^{*} - 1 = \sum_j \omega_{t-1}^j \left(\frac{q_t^j}{q_{t-1}^j} - 1 \right) \quad ; \quad \sum_j \omega_{t-1}^j = 1 .$$

In (11), $\omega_{t-1}^j \left[\left(q_t^j / q_{t-1}^j \right) - 1 \right]$ is BEA's formula for a component's contribution to GDP growth.[6] However, noting (8) and (9), BEA's growth decomposition in the second expression in (11) is *exact* only if relative prices are constant or $r_{t-1}^j = r_t^j = 1$ so that $\sum_j r_t^j w_{t-1}^j = \sum_j \omega_{t-1}^j = 1$. Therefore, if relative prices are *not* constant, $Q_{t-1,t}^{*} - 1 \neq (X_t / X_{t-1}) - 1$ and growth residuals arise from changes in relative prices that BEA does not take into account.

III. Exact Level Aggregation and Growth Decomposition

The preceding analysis showed that to eliminate the residuals in Table 1 relative prices need to be taken into account. This is implemented in the following illustrations.

III-A. Exactly Additive Contributions to GDP Level

The GDP level aggregation in this paper was given earlier by (8),[7]

$$(12) \quad X_t = \frac{Y_t}{P_{0,t}} = \sum_j r_t^j X_t^j = \sum_j \frac{P_{0,t}^j}{P_{0,t}} \frac{Y_t^j}{P_{0,t}^j} = \sum_j \frac{Y_t^j}{P_{0,t}} \quad ; \quad Y_t = \sum_j Y_t^j .$$

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indexes recommended by the UN System of National Accounts 1993 and adopted by Canada and the US.

[6] Moulton and Seskin (1999), p. 16, gives the formula for the weight ω_{t-1}^j that obviously sums to 1 and, thus, implies (11). Another weight formula that looks different but can be shown to be equivalent to ω_{t-1}^j was derived by Dumagan (2002) that according to Balk (2004) is an independent rediscovery of the same weight derived by Van IJzeren (1952).

[7] It may be noted that the application of relative price weights to industry GDP to obtain aggregate GDP in (8) or (12) has already been applied by Dumagan (2013) to industry labor productivity to obtain aggregate labor productivity following the same procedure by Tang and Wang (2004) when GDP is in chained prices that Dumagan generalized to *any* real GDP, i.e., in chained or in constant prices.

From (12), $X_t = \sum_j r_t^j \, X_t^j$ implies that the industry level contribution, $r_t^j X_t^j = Y_t^j / P_{0,t}$, must be additive because the common US deflator means that $r_t^j X_t^j$ is measured in "homogeneous" units and, therefore, has the same real value across industries. This may be made clearer by an analogous example of converting real GDP of countries to "purchasing power parity" (PPP) values to make them additive.

Suppose US nominal GDP is $\$Y^S$ and US GDP deflator is P^S so that US real GDP is $\$Y^S/P^S$. Also, suppose UK nominal GDP is $£Y^K$ and UK GDP deflator is P^K so that UK real GDP is $£Y^K/P^K$. Without the currency denominations, \$ and £, and the deflators, P^S and P^K, then Y^S and Y^K are just "numbers" in which case the *simple* sum, $Y^S + Y^K$, makes sense because "one" of Y^S is the same as "one" of Y^K. However, the simple sum of US and UK real GDP, $\$Y^S/P^S + £Y^K/P^K$, is not sensible because they are not in the same units. For this sum to be sensible, one way is to express the units in US PPP values. This requires multiplying $£Y^K/P^K$ by the "real exchange rate" (RER) as follows,

$$(13) \qquad \frac{\$Y^S}{P^S} + \frac{£Y^K}{P^K}\left(\frac{P^K}{P^S}\right)\left(\frac{\$}{£}\right) = \frac{\$Y^S}{P^S} + \frac{£Y^K}{P^K}\left(\frac{\$/P^S}{£/P^K}\right) = \frac{\$Y^S}{P^S} + \frac{\$Y^K}{P^S}.$$

In (13), $(\$/P^S)/(£/P^K)$ is the RER that adjusts the nominal exchange rate, $\$/£$, for differences in purchasing power, i.e., difference between P^S and P^K. Thus, RER converts UK real GDP to the same units as US real GDP. The end result is that one unit of Y^S and one unit of Y^K have the same exchange value given by $(\$/P^S)/(\$/P^S) = 1$, which demonstrates PPP.[8]

Following the above example, this paper's industry GDP level contribution given by $r_t^j X_t^j = Y_t^j / P_{0,t}$ is a PPP value. In this case, since all *j* are in the *same* country, the nominal exchange rate is $1/1$ and the common deflator, $P_{0,t}$, means that the exchange value between each unit of $r_t^j X_t^j$ is $\left(1/P_{0,t}\right)/\left(1/P_{0,t}\right) = 1$. Thus, all $r_t^j X_t^j$ are PPP values and, therefore, exactly additive across industries, i.e., no residuals (see Table 2). This follows from the fact that $X_t = \sum_j r_t^j \, X_t^j$ implies $Y_t = \sum_j Y_t^j$, which is exactly additive.

It is important to note the generality of this paper's PPP aggregation procedure in (12) so that it applies to real GDP regardless of the deflator formulas. Moreover, the industry deflators

[8] To express (13) in terms of "consumer" PPP, the GDP deflators, P^S and P^K, need only to be replaced by the corresponding US and UK consumer price indexes.

and the aggregate deflator need not have the same functional form because the industry deflators cancel out and only the aggregate deflator is relevant in the aggregation. Hence, PPP values are in chained prices or in constant prices depending on the aggregate deflator.

The preceding analysis implies that *without* the relative price r_t^j, the additivity of $X_t^j = Y_t^j / P_{0,t}^j$ is questionable because X_t^j is not in homogeneous units of measure across industries. Hence, X_t^j is appropriate only in examining an industry in "isolation" since relative prices are irrelevant when there is only one industry. However, once the analysis involves a "group" of industries, relative prices need to be taken into account.

Given the "generality" of PPP conversion by way of (12), the computations in this paper will be referred to as GEN in the following tables to distinguish them from those by BEA. In Table 2, PPP values are also in chained 2009 dollars because these are obtained by $r_t^j X_t^j = Y_t^j / P_{0,t}$ where $P_{0,t}$ is the US GDP chained price index or deflator with 2009 as the base period.

Table 2. Conversion of US GDP in Chained Prices to Exactly Additive PPP Values

	BEA		GEN			
	GDP in Chained Prices (billion chained 2009 dollars)		Relative Prices (weights)		GDP in PPP Values (billion chained 2009 dollars)	
	2011	2012	2011	2012	2011	2012
	(1)	(2)	(3)	(4)	(5) = (1)x(3)	(6) = (2)x(4)
US GDP	**15,052.4**	**15,470.7**	**1.00**	**1.00**	**15,052.4**	**15,470.7**
Industry GDP weighted by relative prices (in PPP values)						
Agriculture, forestry, fishing, and hunting	134.6	135.0	1.423	1.419	191.6	191.5
Mining	301.0	343.3	1.318	1.192	396.6	409.2
Utilities	284.6	289.6	0.953	0.905	271.3	262.0
Construction	549.1	571.2	0.964	0.969	529.2	553.4
Durable goods	1,040.8	1,083.2	0.937	0.937	975.5	1,014.5
Nondurable goods	813.1	808.8	1.092	1.141	887.8	922.8
Wholesale trade	862.1	884.2	1.022	1.037	881.2	916.8
Retail trade	872.0	883.5	0.994	1.000	866.9	883.6
Transportation and warehousing	437.4	442.1	0.992	1.016	433.9	449.1
Information	745.3	777.8	0.964	0.951	718.3	739.7
Finance and insurance	960.5	982.8	1.021	1.045	980.3	1,026.8
Real estate and rental and leasing	1,994.6	2,038.1	0.972	0.979	1,938.1	1,994.6
Professional, scientific, and technical services	1,051.1	1,094.9	0.995	0.992	1,045.7	1,085.9
Management of companies and enterprises	279.8	302.5	0.980	0.969	274.1	293.0
Administrative and waste management services	452.2	468.7	0.992	0.994	448.5	466.1
Educational services	165.0	166.6	1.022	1.042	168.6	173.6
Health care and social assistance	1,072.9	1,101.8	1.002	1.000	1,074.7	1,102.3
Arts, entertainment, and recreation	150.6	154.0	0.967	0.973	145.6	149.8
Accommodation and food services	414.8	426.4	0.964	0.981	399.7	418.3
Other services, except government	322.3	328.3	1.015	1.021	327.0	335.2
Federal government	681.5	674.4	1.017	1.005	692.9	677.8
health care and social assistance	1,390.9	1,394.3	1.010	1.007	1,404.9	1,404.3
Residuals: "Not allocated by industry"	**58.0**	**96.6**			**0**	**0**

Source: Author's calculations by applying this paper's procedure for PPP conversion in (8) or (12) to BEA GDP in chained prices in Table 1.

III-B. Exactly Additive Contributions to GDP Growth

By definition, US GDP growth g_t and industry GDP growth g_t^j are,

$$(14) \quad g_t \equiv \frac{X_t}{X_{t-1}} - 1 \quad ; \quad g_t^j \equiv \frac{X_t^j}{X_{t-1}^j} - 1.$$

Combining (8), (9), and (14), it can be verified that,

$$(15) \quad g_t = \sum\nolimits_j \left[s_{t-1}^j g_t^j + \left(r_t^j - r_{t-1}^j\right) w_{t-1}^j g_t^j + \left(r_t^j - r_{t-1}^j\right) w_{t-1}^j \right].$$

The growth contribution of each industry in (15) is broken out into three components in Table 3 under the heading GEN.[9] These are given by,

(16) PGE (pure growth effect) $= s_{t-1}^j g_t^j$;

(17) GPIE (growth-price interaction effect) $= \left(r_t^j - r_{t-1}^j\right) w_{t-1}^j g_t^j$;

(18) RPE (relative price effect) $= \left(r_t^j - r_{t-1}^j\right) w_{t-1}^j$.

PGE may be interpreted as an industry's growth contribution due to with-in industry efficiency changes, holding relative prices constant so that GPIE and RPE are zero. On the other hand, when there are no efficiency changes so that g_t^j is zero, an industry's growth contribution could come from non-zero RPE when relative prices change and induce resource reallocation between industries.

For comparison, the BEA growth contributions in Table 1 are reproduced in Table 3. For all industries, BEA yields 2.50 percent while GEN yields PGE + GPIE + RPE $= 2.78$ percent, the "actual" GDP growth in 2012. Thus, GEN leaves no growth residuals. Note that for *each* industry, BEA's growth contribution approximately equals PGE. Therefore, BEA almost totally excludes GPIE and RPE, which amounts to ignoring the effects on GDP growth of changes in relative prices. These exclusions could have significant effects, even sign reversals of growth contributions. Table 3 shows two sign reversals: *utilities* and *nondurable goods*. In the latter case, by excluding GPIE and RPE, the growth contribution of *nondurable goods* switches in sign

[9] To breakout the industry growth contributions in (15) into PGE, GPIE, and RPE, note that (7) implies $s_{t-1}^j = r_{t-1}^j w_{t-1}^j$. Hence, $\sum_j s_{t-1}^j = \sum_j r_{t-1}^j w_{t-1}^j = 1$ may be used while $\sum_j s_{t-1}^j g_t^j$ may be added and $\sum_j r_{t-1}^j w_{t-1}^j g_t^j$ may be subtracted simultaneously in the right-hand side.

from positive (0.23) according to GEN to negative (− 0.03) according to BEA. Hence, excluding the effects of changes in relative prices could make BEA's growth contributions misleading.

Table 3. Exactly Additive Industry Contributions to US GDP Growth

	BEA	GEN			
	GDP growth	**PGE**	**GPIE**	**RPE**	**GDP growth**
	(percent)	(percent)	(percent)	(percent)	(percent)
	2012	2012	2012	2012	2012
		(1)	(2)	(3)	(1)+(2)+(3)
Industry Contributions to US GDP growth (percentage point)					
Agriculture, forestry, fishing, and hunting	0.00	0.004	0.000	-0.004	0.000
Mining	0.35	0.370	-0.035	-0.251	0.084
Utilities	**0.03**	**0.032**	**-0.002**	**-0.092**	**-0.062**
Construction	0.14	0.141	0.001	0.019	0.161
Durable goods	0.26	0.264	0.000	-0.004	0.259
Nondurable goods	**-0.03**	**-0.031**	**-0.001**	**0.265**	**0.233**
Wholesale trade	0.15	0.150	0.002	0.084	0.237
Retail trade	0.08	0.076	0.000	0.035	0.111
Transportation and warehousing	0.03	0.031	0.001	0.069	0.101
Information	0.21	0.208	-0.003	-0.063	0.142
Finance and insurance	0.15	0.151	0.004	0.155	0.309
Real estate and rental and leasing	0.28	0.281	0.002	0.093	0.375
Professional, scientific, and technical services	0.29	0.289	-0.001	-0.021	0.267
Management of companies and enterprises	0.15	0.148	-0.002	-0.020	0.126
Administrative and waste management services	0.11	0.109	0.000	0.008	0.117
Educational services	0.01	0.011	0.000	0.022	0.033
Health care and social assistance	0.19	0.192	0.000	-0.009	0.183
Arts, entertainment, and recreation	0.02	0.022	0.000	0.006	0.028
Accommodation and food services	0.07	0.074	0.001	0.048	0.123
Other services, except government	0.04	0.040	0.000	0.014	0.054
Federal government	-0.05	-0.048	0.001	-0.053	-0.101
State and local government	0.02	0.023	0.000	-0.027	-0.004
Sum	**2.50**	**2.54**	**-0.03**	**0.27**	**2.78**
US GDP percent growth	**2.78**				**2.78**
Residuals: "Not allocated by industry"	**0.28**				**0.00**

Source: BEA publishes growth contributions only up to two decimal places as shown above (reproduced from Table 1). The results under the heading GEN are the author's calculations of exactly additive industry growth contributions broken out into *pure growth effect* (PGE) in (16), *growth-price interaction effect* (GPIE) in (17), and *relative price effect* (RPE) in (18).

III-C. Consistent Aggregation of Implicit Indexes

Consistent aggregation means that X_t^j/X_{t-1}^j may be calculated at different levels of aggregation while allowing w_{t-1}^j and r_t^j to adjust to *maintain* the value of $X_t/X_{t-1} = \sum_j r_t^j\, w_{t-1}^j(X_t^j/X_{t-1}^j)$. Table 4 shows $X_t/X_{t-1} = 15{,}470.7/15{,}052.4 = 1.0278$ is maintained while the number of industries is changed from fifteen to twenty-two.

It appears in Table 4 that the implicit US GDP Fisher quantity index is consistent in aggregation by the fact that the index value of 1.0278 for 2012 remains the same–implying that the GDP growth of 2.78 percent also remains the same–when the number of implicit industry GDP Fisher quantity indexes being aggregated changes from twenty-two to fifteen industries. This result generalizes to any finite number of industries.

Table 4. Consistent Aggregation of the Implicit US GDP Fisher Quantity Index

	Twenty-two industries		**Fifteen industries**	
	Implicit Fisher quantity indexes	Weighted Implicit Fisher quantity indexes	Implicit Fisher quantity indexes	Weighted Implicit Fisher quantity indexes
	2012	**2012**	**2012**	**2012**
Gross domestic product	**1.0278**		**1.0278**	
Agriculture, forestry, fishing, and hunting	1.0030	0.0127	1.0030	0.0127
Mining	1.1405	0.0272	1.1405	0.0272
Utilities	1.0176	0.0174	1.0176	0.0174
Construction	1.0402	0.0368	1.0402	0.0368
Manufacturing			1.0185	0.1287
Durable goods	1.0407	0.0674		
Nondurable goods	0.9947	0.0613		
Wholesale trade	1.0256	0.0609	1.0256	0.0609
Retail trade	1.0132	0.0587	1.0132	0.0587
Transportation and warehousing	1.0107	0.0298	1.0107	0.0298
Information	1.0436	0.0491	1.0436	0.0491
Finance, insurance, real estate, rental, and leasing			1.0223	0.2007
Finance and insurance	1.0232	0.0682		
Real estate and rental and leasing	1.0218	0.1325		
Professional and business services			1.0464	0.1226
Professional, scientific, and technical services	1.0417	0.0721		
Management of companies and enterprises	1.0811	0.0195		
Administrative and waste management services	1.0365	0.0310		
Edu. services, health care, and social assistance			1.0246	0.0848
Educational services	1.0097	0.0115		
Health care and social assistance	1.0269	0.0732		
Arts, entertainment., rec., accom., and food services			1.0265	0.0377
Arts, entertainment, and recreation	1.0226	0.0100		
Accommodation and food services	1.0280	0.0278		
Other services, except government	1.0186	0.0223	1.0186	0.0223
Government			0.9983	0.1383
Federal government	0.9896	0.0450		
State and local government	0.9936	0.0933		
Sum of weighted Fisher subaggregate quantity indexes		**1.0278**		**1.0278**

Source: Author's calculations from the index aggregation procedure in (8) applied to data in Table 1. The GDP for the subaggregates in bold italics above are not included in Table 1 but are readily available from the BEA website.

IV. Summary and Conclusion

The GDP level aggregation procedure in this paper is by application of relative prices as weights of industry real GDP to convert them into PPP values that are in homogeneous units of measure for additivity. In turn, this leads to GDP growth decomposition that permits separating industry growth contributions into *pure growth effects* due to with-in industry efficiency changes, holding relative prices constant, and to *growth-price interaction* and *relative price effects* that, even when there are no efficiency changes, induce resource reallocation between industries.

The above procedures are exact in that the sums of level contributions and growth contributions of industries equal, respectively, the "actual" GDP level and growth. These procedures make clear that the residuals in US industry level and growth contributions are due to effects of differences and changes in relative prices ignored by BEA. Consequently, BEA's growth contributions are inexact and, thus, could be misleading. However, once these relative price effects are correctly taken into account, the residuals disappear.

In sum, by applying relative price weights to convert GDP–either in chained prices or in constant prices–to PPP values that are exactly additive across industries, the procedures in this paper ensure consistent level aggregation and growth decomposition of *any* real GDP.

References

Aspden, Charles, 2000. "Introduction of Chain Volume and Price Measures-the Australian Approach, " Paper presented at the Joint ADB/ESCAP Workshop on Rebasing and Linking of National Accounts Series, Bangkok, Thailand (March 21-24).

Balk, Bert M., 1996. "Consistency-in-Aggregation and Stuvel Indices," *Review of Income and Wealth*, 42: 353-363.

Balk, Bert M., 2004. "Decompositions of Fisher Indexes," *Economics Letters*, 82: 107-113.

Brueton, Anna, 1999. "The Development of Chain-linked and Harmonized Estimates of GDP at Constant Prices," *Economic Trends*, 552: 39-45, Office for National Statistics, UK.

Chevalier, Michel, 2003. "Chain Fisher Volume Index Methodology," Income and Expenditure Accounts, Technical Series, Statistics Canada. Ottawa, Ontario K1A 0T6.

Diewert, W. Erwin, 1978. "Superlative Index Numbers and Consistency in Aggregation," *Econometrica*, 46: 883-900.

Dumagan, Jesus C., 2002. "Comparing the Superlative Törnqvist and Fisher Ideal Indexes," *Economics Letters*, 76: 251-258.

Dumagan, Jesus C., 2013. "A Generalized Exactly Additive Decomposition of Aggregate Labor Productivity Growth," *Review of Income and Wealth*, 59 (Issue 1): 157-168. An earlier version was circulated as *Discussion Paper Series* No. 2011-19, Makati City: Philippine Institute for Development Studies.

Ehemann, Christian, Arnold J. Katz and Brent R. Moulton, 2002. "The Chain-Additivity Issue and the US National Economic Accounts," *Journal of Economic and Social Measurement*, 28: 37-49.

European Union, 2007. "Changes to National Accounts in 2005 and Introduction of Chain-Linking into National Accounts," (Status report as of October 12, 2007), available at www.europa.eu.int/estatref/info/sdds/en/na/na_changes2005.pdf.

Fisher, Irving M., 1922. *The Making of Index Numbers*, Boston: Houghton Mifflin Co.

Landefeld, J. Steven and Robert P. Parker, 1997. "BEA's Chain Indexes, Time Series, and Measures of Long-Term Economic Growth," *Survey of Current Business*, 77: 58-68.

Maruyama, Masaaki, 2005. "Japan's Experience on the Chain-Linking Method and on Combining Supply-Side and Demand-Side Data for Quarterly GDP," Paper presented at the 9th NBS/OECD Workshop on National Accounts, Xiamen, China.

Moulton, Brent R. and Eugene P. Seskin, 1999. "A Preview of the 1999 Comprehensive Revision of the National Income and Product Accounts," *Survey of Current Business*, 79: 6-17.

Schreyer, Paul, 2004. "Chain Index Number Formulae in the National Accounts," Paper presented at the 8th OECD–NBS Workshop on National Accounts, Paris, France.

Seskin, Eugene P. and Robert P. Parker, 1998. "A guide to the NIPA's," *Survey of Current Business*, 78: 26-68.

Tang, Jianmin and Weimin Wang, 2004. "Sources of Aggregate Labour Productivity Growth in Canada and the United States," *The Canadian Journal of Economics* 37 (2): 421-44.

Van IJzeren, J., 1952. “Over de Plausibiliteit van Fisher’s Ideale Indices (On the Plausibility of Fisher’s Ideal Indices),” *Statistische en Econometrische Onderzoekingen (C.B.S.). Nieuwe Reeks*, 7: 104-115.

Vartia, Yrjo O., 1976. “Ideal Log Change Index Numbers,” *Scandinavian Journal of Statistics*, 3: 121-126.

Whelan, Karl, 2002. “A guide to US Chain-Aggregated NIPA Data,” *Review of Income and Wealth*, 48: 217-233.