An Alternative Framework for Sectoral Contributions to GDP Level and Growth: Application to the Philippines[§]

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This paper applies *relative price* weights–where relative price is the ratio of a sector's GDP deflator to the aggregate GDP deflator–to convert sectoral real GDP to homogeneous units using the economy's GDP as "numeraire" in an alternative framework for GDP level aggregation and growth decomposition. This alternative and the "traditional" framework–without relative price weights–are compared and applied to Philippine GDP to show that the latter framework is deficient and misleading for its inability to determine the effects on GDP growth of changes and differences in sectoral relative prices that need to be taken into account.

Key Words: Real GDP; relative prices; index numbers; aggregation; additivity

JEL classification: C43, O47

I. Introduction

GDP deflators differ between sectors (e.g., industries) and also differ from the economy-wide GDP deflator and, thus, it is arguable that *real* GDP of industries are not necessarily measured in *homogeneous* units and, therefore, their simple addition, i.e., without weights, in current practice is questionable. Hence, in group analysis involving industries, real GDP of the industries need conversion to homogeneous units for consistent aggregation. Therefore, this paper applies *relative price* weights–where relative price is the ratio of an industry's GDP deflator to the aggregate GDP deflator–to convert real GDP of industries to homogeneous units using the economy's GDP as "numeraire" in an alternative framework for GDP level aggregation and growth decomposition. For comparison, this alternative and the "traditional" framework–

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without relative price weights–are applied to Philippine GDP to show that the latter framework is deficient and misleading for its inability to determine the effects of changes and differences in industry relative prices on GDP growth that theory implies exist and, thus, need to be known.

The rest of this paper is organized as follows. Section II presents GDP value index decomposition employing either *direct* indexes for GDP in constant prices or *chained* indexes for GDP in chained prices. The end result is a "generalized" (GEN, in short) framework for GDP level aggregation and growth decomposition that applies to *any* real GDP. In this GEN framework, relative price weights *resolve* the "non-additivity" of industry GDP in chained prices by converting them to homogeneous units for additivity with GDP as the numeraire. Similarly, relative price weights *reformulate* the "additivity" of industry GDP in constant prices also by converting them to homogeneous units. Section III re-examines the "traditional" (TRAD, in short) framework for level aggregation and growth decomposition of GDP in constant prices and compares it to this paper's GEN framework. Section IV presents a comparative application of TRAD and GEN to Philippine GDP in constant prices.[1] Section V concludes this paper.

II. "Generalized" (GEN) GDP Level Aggregation and Growth Decomposition

Readily available data on *nominal* GDP (i.e., in current prices) and on *real* GDP (i.e., either in constant prices or in chained prices), by sectors or industries over time, will suffice for the analytic procedures in this paper. The price and quantity indexes used in the analysis are *implicit* or computed values obtained from the above nominal and real values. Thus, data on commodity prices and quantities are not necessary.

For analytical purposes, let 0 be the base period that is more than one period away from $t-1$ and t. To link period 0 to $t-1$ and t, denote the price indexes by $P_{0,t-1}$ and $P_{0,t}$; and the quantity indexes by $Q_{0,t-1}$ and $Q_{0,t}$. The GDP value index–i.e., relative change of GDP in current prices from Y_0 to Y_{t-1} or to Y_t–can be expressed as the product of pairs of price and quantity indexes. That is,

$$(1)\quad \frac{Y_{t-1}}{Y_0} = P_{0,t-1} \times Q_{0,t-1} \quad ; \quad \frac{Y_t}{Y_0} = P_{0,t} \times Q_{0,t} \, .$$

[1] This GEN framework was applied to US GDP in chained prices in Dumagan (2014).

Two pairs of price and quantity indexes that satisfy (1) are employed in current practice of computing real GDP. One pair is the Paasche price and Laspeyres quantity indexes and the other is the Fisher price and Fisher quantity indexes (Fisher, 1922; Balk, 2010).

The expressions in (1) yield aggregate *real* GDP defined by,

$$(2) \quad X_{t-1} \equiv \frac{Y_{t-1}}{P_{0,t-1}} = Y_0 \times Q_{0,t-1} \quad ; \quad X_t \equiv \frac{Y_t}{P_{0,t}} = Y_0 \times Q_{0,t} \,.$$

Moreover, by similar procedures, industry *real* GDP is also computed for each *j* by,

$$(3) \quad X_{t-1}^j \equiv \frac{Y_{t-1}^j}{P_{0,t-1}^j} = Y_0^j \times Q_{0,t-1}^j \quad ; \quad X_t^j \equiv \frac{Y_t^j}{P_{0,t}^j} = Y_0^j \times Q_{0,t}^j \,.$$

It may be noted that (2) and (3) yield *implicit* price and quantity indexes from the real and nominal values of aggregate and industry GDP in published national accounts data.

In current practice, real GDP in (2) and (3) are in *constant* prices if the Paasche price and Laspeyres quantity index are *direct* (e.g., in the Philippines and other "developing" countries) but are in *chained* prices if these indexes are *chained* (e.g., Australia, Japan, UK, and other "developed" countries in the EU and OECD). Alternatively, *chained* Fisher price and Fisher quantity indexes are used to obtain GDP in chained prices but only in Canada and the US.[2]

The second expressions in (2) and (3) yield GDP in constant prices where,

$$(4) \quad X_t \equiv \frac{Y_t}{P_{0,t}} \quad ; \quad X_t^j \equiv \frac{Y_t^j}{P_{0,t}^j} \,;$$

$$(5) \quad X_t = \sum_j X_t^j \quad \text{if } P_{0,t} \text{ and } P_{0,t}^j \text{ in (4) are } \mathit{direct} \text{ Paasche price indexes.}$$

The result in (5) shows "additivity" of GDP in constant prices (see the Appendix) based on the "consistency-in-aggregation" property of the Paasche formula (Balk, 2010; Vartia, 1976). In contrast, (2) and (3) yield GDP in *chained* prices where,

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[2] For country practices of GDP in chained prices, see Aspden (2000) for Australia; Maruyama (2005) for Japan; Brueton (1999) for the UK; Schreyer (2004) and EU (2007) for EU and OECD countries; Chevalier (2003) for Canada; and Landefeld and Parker (1997), also Moulton and Seskin (1999) for the US. Brueton (1999) noted that the EU System of National Accounts 1995 recommended Paasche price and Laspeyres quantity indexes as more practical than the theoretically superior Fisher price and Fisher quantity indexes recommended by the UN System of National Accounts 1993 and adopted by Canada and the US.

$$(6) \quad X_t \equiv \frac{Y_t}{P_{0,t}} \quad ; \quad X_t^j \equiv \frac{Y_t^j}{P_{0,t}^j} \;;$$

$$(7) \quad X_t \neq \sum_j X_t^j \;\text{ if } P_{0,t} \text{ and } P_{0,t}^j \text{ in (6) are } \textit{chained} \text{ Paasche price or Fisher price indexes.}$$

The result in (7) shows "non-additivity" of GDP in chained prices (Balk, 2010; Ehemann, Katz, and Moulton, 2002; Whelan, 2002).[3]

However, this paper formulates a framework for GDP level aggregation and growth decomposition that is *exactly* additive for GDP either in constant prices or in chained prices, i.e., regardless of the formula for $P_{0,t}$ and $P_{0,t}^j$. For this purpose, note that (4) to (7) satisfy,

$$(8) \quad Y_t = \sum_j Y_t^j \quad ; \quad s_t^j \equiv \frac{Y_t^j}{Y_t} \quad ; \quad \sum_j s_t^j = 1.$$

Industry relative prices–defined as ratios of industry GDP deflators to the aggregate GDP deflator–play a crucial role in this paper's framework. Also, this framework uses the ratios of industry real GDP to aggregate real GDP. Hence, define,[4]

$$(9) \quad r_t^j \equiv \frac{P_{0,t}^j}{P_{0,t}} \quad ; \quad w_t^j \equiv \frac{X_t^j}{X_t} \quad ; \quad r_t^j = \frac{Y_t^j/X_t^j}{Y_t/X_t} = \frac{Y_t^j/Y_t}{X_t^j/X_t} = \frac{s_t^j}{w_t^j}.$$

Note that all the above relations hold also at t-1. Therefore, regardless of the index formula for $P_{0,t}$ and $P_{0,t}^j$, it follows that by applying the relative price weights, r_t^j, to industry real GDP the general result from (2) to (9) is that,[5]

$$(10) \quad \frac{Y_t}{P_{0,t}} = \sum_j \frac{P_{0,t}^j}{P_{0,t}} \frac{Y_t^j}{P_{0,t}^j} \quad ; \quad X_t = \sum_j r_t^j X_t^j \quad ; \quad \frac{X_t}{X_{t-1}} = \sum_j r_t^j w_{t-1}^j \frac{X_t^j}{X_{t-1}^j} \;;$$

$$(11) \quad \sum_j r_t^j w_{t-1}^j \neq 1 \;\text{ if }\; r_{t-1}^j \neq r_t^j \quad ; \quad \sum_j r_t^j w_{t-1}^j = \sum_j w_{t-1}^j = 1 \;\text{ only if }\; r_{t-1}^j = r_t^j = 1.$$

[3] Non-additivity is universal in countries that have implemented GDP in chained prices.

[4] This paper avoids calling w_t^j as "real shares"–that usually connote that they sum to one–because w_t^j do not necessarily sum to one in the analytical framework of this paper.

[5] The application of relative price weights to industry GDP to obtain aggregate GDP in (10) was implemented by Dumagan (2013) to labor productivities of industries to obtain aggregate labor productivity following the same procedure by Tang and Wang (2004) for GDP in chained prices that Dumagan generalized to *any* real GDP, i.e., in chained or in constant prices.

Real GDP, X_t, is either in constant or in chained prices depending on the aggregate deflator, $P_{0,t}$, since the industry deflators, $P_{0,t}^j$, cancel out when (10) is simplified.[6] Moreover, (10) and (11) show that X_t equals the weighted sum of X_t^j and the *implicit* GDP quantity index, X_t/X_{t-1}, equals the weighted sum of the *implicit* industry quantity indexes, X_t^j/X_{t-1}^j, where the sum of the weights may not equal 1 unless all prices change proportionately (i.e., constant relative prices), in which case all price indexes are equal so that $r_{t-1}^j = r_t^j = 1$.[7] Note that X_t^j/X_{t-1}^j may be calculated at different levels of aggregation while allowing w_{t-1}^j and r_t^j to adjust to *maintain* the value of X_t/X_{t-1}.[8] Thus, X_{t-1}, X_t, and X_t/X_{t-1} are consistent in aggregation.

This paper's aggregation procedure in (10) *reformulates* the "additivity" of GDP in constant prices and *resolves* the "non-additivity" of GDP in chained prices considering that the *common* GDP deflator, $P_{0,t}$, converts GDP to homogeneous units that are necessarily additive. In this case, GDP of *all* sectors or industries are converted to homogeneous units with GDP as numeraire, measured either in constant or in chained prices depending on the formula for $P_{0,t}$. Hence, (10) and (11) comprise a "generalized" (GEN) framework for *any* real GDP. In this light, all procedures from this framework will be referred to as GEN, in short, for expository purposes.

II-A. GEN GDP Level Aggregation by "Purchasing Power Parity" Conversion

In the GEN aggregation equation, $X_t = \sum_j r_t^j X_t^j$, industry level contributions, $r_t^j X_t^j = Y_t^j/P_{0,t}$, are additive because they are measured in homogeneous units or in the same real

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[6] This result implies that the industry deflators and the aggregate deflator need not have the same functional form because the industry deflators cancel out and only the aggregate deflator is relevant in the aggregation.

[7] Strictly speaking, r_t^j is a "relative price index" since it is the ratio of price indexes, the industry GDP deflator divided by the aggregate GDP deflator. In contrast, "relative price" is the ratio of commodity prices. By construction of price indexes, when all commodity prices change in the same proportion, e.g., $\delta\%$, then relative prices are the same and all price indexes for industries and for the aggregate equal $(1 + \delta/100)$ so that $r_t^j = 1$. If the proportionate change remains $\delta\%$ in other years, i.e., constant, then $r_{t-1}^j = r_t^j = 1$.

[8] Dumagan (2014) provides an empirical illustration of this result for the implicit US GDP Fisher quantity indexes.

value per unit across industries. This may be clarified by an analogous example of converting real GDP of countries to "purchasing power parity" (PPP) values to make them additive.

Suppose US nominal GDP is $\$Y^S$ and US GDP deflator is P^S so that US real GDP is $\$Y^S/P^S$. Also, suppose Philippine (PH) nominal GDP is ₱Y^P and PH GDP deflator is P^P so that PH real GDP is ₱Y^P/P^P. Without the currency denominations, \$ and ₱, and the deflators, P^S and P^P, then Y^S and Y^P are just "numbers" in which case the *simple* sum, $Y^S + Y^P$, makes sense because "one" of Y^S is the same as "one" of Y^P. But the simple sum, $\$Y^S/P^S +$ ₱Y^P/P^P, is not sensible because they are not in the same units. For this sum to be sensible, one way is to express the units in US PPP values. This requires multiplying ₱Y^P/P^P by the "real exchange rate" (RER) as follows,

$$(12)\qquad \frac{\$Y^S}{P^S} + \frac{₱Y^P}{P^P}\left(\frac{P^P}{P^S}\right)\left(\frac{\$}{₱}\right) = \frac{\$Y^S}{P^S} + \frac{₱Y^P}{P^P}\frac{(\$/P^S)}{(₱/P^P)} = \frac{\$Y^S}{P^S} + \frac{₱Y^P(\$/₱)}{P^S}.$$

In (12), $(\$/P^S)/(₱/P^P)$ is the RER that adjusts the nominal exchange rate, $\$/₱$, for differences in purchasing power, i.e., difference between P^S and P^P. Thus, RER converts PH real GDP to the same units as US real GDP. The end result is that one unit of $\$Y^S/P^S$ and one unit of ₱$Y^P(\$/₱)/P^S$ are the same in real US\$, with exchange value of $(1/P^S)/(1/P^S) = 1$, which demonstrates PPP.[9]

Following the above example, it can be seen that $r_t^j X_t^j = Y_t^j/P_{0,t}$ is a PPP value. In this case, since all j are in the *same* country, the nominal exchange rate is $1/1$ and the common deflator, $P_{0,t}$, means that the exchange value between each unit of Y_t^j is $(1/P_{0,t})/(1/P_{0,t}) = 1$. Thus, all $r_t^j X_t^j$ are PPP values and, therefore, exactly additive from the fact that $X_t = \sum_j r_t^j X_t^j$ implies $Y_t = \sum_j Y_t^j$, which is exactly additive.

II-B. GEN GDP Growth Decomposition

By definition, GDP growth g_t and industry GDP growth g_t^j are,

$$(13)\qquad g_t \equiv \frac{X_t}{X_{t-1}} - 1 \quad ; \quad g_t^j \equiv \frac{X_t^j}{X_{t-1}^j} - 1\,.$$

[9] To express (12) in "consumer" PPP, the GDP deflators, P^S and P^P, need to be replaced by the corresponding US and PH consumer price indexes.

Combining (10), (11), and (13), it can be verified that,[10]

(14) $$g_t = \sum_j \left[s_{t-1}^j g_t^j + \left(r_t^j - r_{t-1}^j\right) w_{t-1}^j g_t^j + \left(r_t^j - r_{t-1}^j\right) w_{t-1}^j\right].$$

In (14), the growth contribution of each industry is broken out into three components,

(15) PGE (pure growth effect) $= s_{t-1}^j g_t^j$;

(16) GPIE (growth-price interaction effect) $= \left(r_t^j - r_{t-1}^j\right) w_{t-1}^j g_t^j$;

(17) RPE (relative price effect) $= \left(r_t^j - r_{t-1}^j\right) w_{t-1}^j$.

PGE may be interpreted as an industry's growth contribution due to with-in industry efficiency changes, holding relative prices constant so that GPIE and RPE are zero. On the other hand, when there are no efficiency changes so that PGE is zero, an industry's growth contribution could come from non-zero GPIE and RPE when relative prices change and induce resource reallocation between industries.

III. "Traditional" (TRAD) GDP Level Aggregation and Growth Decomposition

The TRAD framework for GDP in constant prices may now be re-examined.

III-A. TRAD GDP Level Aggregation

In TRAD aggregation, GDP in constant prices is the *simple* sum of the GDP in constant prices of industries given earlier in (5) where,

(18) $$X_t = \sum_j X_t^j .$$

However, this paper's GEN aggregation in (10) questions the additivity of $X_t^j = Y_t^j / P_{0,t}^j$ in (18) because X_t^j is not necessarily in homogeneous units of measure because of differences between the industry deflators, $P_{0,t}^j$.[11] However, this question is put aside for the moment to illustrate TRAD growth decomposition.

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[10] To obtain (14), note that (9) implies $s_{t-1}^j = r_{t-1}^j w_{t-1}^j$. Therefore, $\sum_j s_{t-1}^j = \sum_j r_{t-1}^j w_{t-1}^j = 1$ so that either sum may be used in place of 1 if needed. Also, $\sum_j s_{t-1}^j g_t^j$ may be added and $\sum_j r_{t-1}^j w_{t-1}^j g_t^j$ may be subtracted simultaneously in the right-hand side of (14).

[11] Let $j = (1, 2)$ so that $X_t^1 = Y_t^1 / P_{0,t}^1$ and $X_t^2 = Y_t^2 / P_{0,t}^2$ where a unit of Y_t^1 and a unit of Y_t^2 are the same because they are in current prices. However, a unit of X_t^1 is worth $1/P_{0,t}^1$ and a

III-B. TRAD GDP Growth Decomposition

TRAD GDP level aggregation in (18) and the definitions of GDP growth, g_t, and industry GDP growth, g_t^j, in (13) together yield the TRAD GDP growth decomposition,

$$(19) \quad g_t = \sum_j w_{t-1}^j \, g_t^j \quad ; \quad w_{t-1}^j \equiv \frac{X_{t-1}^j}{X_{t-1}} \quad ; \quad \sum_j w_{t-1}^j = 1 \, .$$

It follows from the definitions in (9) that (19) may be rewritten as,

$$(20) \quad g_t = \sum_j w_{t-1}^j \, g_t^j = \sum_j \left(\frac{s_{t-1}^j}{r_{t-1}^j} \right) g_t^j \, .$$

If GDP is in constant prices, it can be shown that the TRAD growth decomposition in (20) and the GEN growth decomposition in (14) will yield the same GDP growth. That is,

$$(21) \quad g_t = \sum_j \left(\frac{s_{t-1}^j}{r_{t-1}^j} \right) g_t^j = \sum_j \left[s_{t-1}^j g_t^j + \left(r_t^j - r_{t-1}^j \right) w_{t-1}^j g_t^j + \left(r_t^j - r_{t-1}^j \right) w_{t-1}^j \right] .$$

However, it can also be shown that the TRAD and GEN growth contributions of the same industry are not equal unless all prices change proportionately, i.e.,

$$(22) \quad \left(\frac{s_{t-1}^j}{r_{t-1}^j} \right) g_t^j \neq s_{t-1}^j g_t^j + \left(r_t^j - r_{t-1}^j \right) w_{t-1}^j g_t^j + \left(r_t^j - r_{t-1}^j \right) w_{t-1}^j \, .$$

By construction of price indexes, *all* price indexes are equal if all prices change in the same proportion so that $r_{t-1}^j = r_t^j = 1$ (see footnote 7) and (22) yields TRAD and GEN contributions of the same industry equal to $s_{t-1}^j g_t^j$. Therefore, if prices do not change in the same proportion, TRAD and GEN will yield unequal growth contributions for the same industry although the sum of industry growth contributions will still equal GDP growth, g_t, in (21).

By taking into account changes in relative prices, the GEN growth contribution of an industry is given by the sum of the three components in the right-hand side of (22). In contrast, TRAD *ignores* changes in relative prices so that the growth contribution of an industry is given only by the single component in the left-hand side. Thus, it appears that TRAD is deficient for its inability to determine the "reallocation effects" on GDP growth of changes and differences in relative prices that are measured by the GPIE and RPE components of GEN. Moreover, TRAD

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unit of X_t^2 is worth $1/P_{0,t}^2$. Since $P_t^1 \neq P_t^2$, the exchange value between X_t^1 and X_t^2 is $\left(1/P_{0,t}^1\right)/\left(1/P_{0,t}^2\right) \neq 1$, implying X_t^1 and X_t^2 are not PPP values and, hence, may not be added.

takes into account the *level* of relative price in an industry's growth contribution but in a *perverse* way. To see this, note that the TRAD growth contribution may be rewritten as,

$$(23) \quad w_{t-1}^{j} g_t^j = \left(\frac{s_{t-1}^j}{r_{t-1}^j}\right) g_t^j = s_{t-1}^j \left(\frac{P_{0,t-1}}{P_{0,t-1}^j}\right) g_t^j .$$

It appears from (23) that TRAD tends to *increase* or overestimate the growth contribution of an industry with a *low* relative price, i.e., $r_{t-1}^j \equiv P_{0,t-1}^j / P_{0,t-1} < 1$. Conversely, TRAD tends to *decrease* or underestimate the growth contribution of an industry with a *high* relative price, i.e., $r_{t-1}^j \equiv P_{0,t-1}^j / P_{0,t-1} > 1$. These results appear contrary to the basic production principle that resources tend to be reallocated to industries with rising relative prices and, thus, promote their growth and raise their contributions to the economy's growth.

IV. Comparative Application of GEN and TRAD to the Philippines

A comparative application of the GEN and TRAD frameworks to Philippine GDP will be instructive. For this purpose, consider Philippine GDP in Table 1.

Table 1. Philippine GDP

	GDP in Current Prices (million current pesos)		GDP in Constant Prices (million constant 2000 pesos)		GDP growth (percent)
	2012	2013	2012	2013	2013
Philippines	**10,567,338**	**11,548,193**	**6,312,173**	**6,765,458**	**7.18**
Agriculture and forestry	1,056,988	1,097,504	568,935	575,616	1.17
Fishing	193,652	199,320	130,032	131,002	0.75
Mining and Quarrying	121,435	115,460	72,046	72,895	1.18
Manufacturing	2,170,918	2,355,416	1,395,711	1,538,913	10.26
Construction	633,066	722,711	348,262	381,656	9.59
Electricity Gas and Water Supply	374,531	400,234	215,423	225,970	4.90
Transport Communication and Storage	685,251	730,023	482,094	509,086	5.60
Trade and Repair of Vehicles, Personal, and Household Goods	1,870,556	2,052,403	1,055,672	1,115,502	5.67
Financial Intermediation	763,670	885,136	426,787	480,683	12.63
Real Estate Renting and Business Activity	1,220,726	1,372,577	678,899	737,938	8.70
Public Administration, Defense, and Social Security	457,620	491,262	274,870	285,377	3.82
Other Services	1,018,925	1,126,147	663,442	710,820	7.14

Source: Economic and Social Database (08-28-2014), Philippine Institute for Development Studies, compiled from the National Accounts, Gross Domestic Product by Industrial Origin (Revised/Rebased), National Statistical Coordination Board.

From Table 1, GEN computes industry relative prices and applies them as weights of industry GDP in constant prices to convert the latter to PPP values that are also in constant prices since the deflator, $P_{0,t}$, is a direct Paasche price index. The results are shown in Table 2 where relative prices change from year to year and differ between industries. Hence, TRAD and GEN growth contributions of the same industry are unequal as shown later in Table 4.

Table 2. GEN Conversion of Philippine GDP in Constant Prices to PPP

	TRAD		GEN			
	GDP in Constant Prices (million constant 2000 pesos)		Relative Prices (weights)		GDP in PPP Values (million constant 2000 pesos)	
	2012	2013	2012	2013	20012	2013
	(1)	(2)	(3)	(4)	(5) = (1) x (3)	(6) = (2) x (4)
Philippines	**6,312,173**	**6,765,458**	**1.000**	**1.000**	**6,312,173**	**6,765,458**
Agriculture and forestry	568,935	575,616	1.110	1.117	631,369	642,968
Fishing	130,032	131,002	0.890	0.891	115,674	116,771
Mining and Quarrying	72,046	72,895	1.007	0.928	72,537	67,642
Manufacturing	1,395,711	1,538,913	0.929	0.897	1,296,751	1,379,910
Construction	348,262	381,656	1.086	1.109	378,148	423,397
Electricity Gas and Water Supply	215,423	225,970	1.039	1.038	223,718	234,475
Transport Communication and Storage	482,094	509,086	0.849	0.840	409,320	427,681
Trade and Repair of Vehicles, Personal, and Household Goods	1,055,672	1,115,502	1.058	1.078	1,117,337	1,202,391
Financial Intermediation	426,787	480,683	1.069	1.079	456,162	518,553
Real Estate Renting and Business Activity	678,899	737,938	1.074	1.090	729,175	804,118
Public Administration, Defense, and Social Security	274,870	285,377	0.994	1.009	273,350	287,804
Other Services	663,442	710,820	0.917	0.928	608,633	659,748

Source: Author's calculations by applying this paper's GEN procedures for PPP in (10) to GDP in Table 1. Industry relative price, the ratio of an industry GDP deflator to the aggregate GDP deflator, is the value per unit of industry real GDP measured in units of aggregate real GDP, the numeraire. Hence, PPP values are "homogeneous" units of measure and. therefore, additive across industries in each column (5) and (6).

Unless GDP is converted to PPP, TRAD "shares of GDP in constant prices"–given by X_t^j/X_t–are unwarranted. The "correct" shares are obtained from $\left(Y_t^j/P_{0,t}\right)/\left(Y_t/P_{0,t}\right)$, which equal industry shares of GDP in current prices, Y_t^j/Y_t. These shares are shown in Table 3.

Table 3. Shares of Philippine GDP

	GDP in Currrent Prices (percent shares)		TRAD GDP in Constant Prices (percent shares)		GEN GDP in PPP Values (percent shares)	
	2012	2013	2012	2013	2012	2013
Philippines	**100.00**	**100.00**	**100.00**	**100.00**	**100.00**	**100.00**
AGRICULTURE, HUNTING, FORESTRY, AND FISHING SECTOR	**11.83**	**11.23**	**11.07**	**10.44**	**11.83**	**11.23**
Agriculture and forestry	10.00	9.50	9.01	8.51	10.00	9.50
Fishing	1.83	1.73	2.06	1.94	1.83	1.73
INDUSTRY SECTOR	**31.23**	**31.12**	**32.18**	**32.81**	**31.23**	**31.12**
Mining and Quarrying	1.15	1.00	1.14	1.08	1.15	1.00
Manufacturing	20.54	20.40	22.11	22.75	20.54	20.40
Construction	5.99	6.26	5.52	5.64	5.99	6.26
Electricity Gas and Water Supply	3.54	3.47	3.41	3.34	3.54	3.47
SERVICE SECTOR	**56.94**	**57.65**	**56.74**	**56.75**	**56.94**	**57.65**
Transport Communication and Storage	6.48	6.32	7.64	7.52	6.48	6.32
Trade and Repair of Vehicles, Personal, and Household Goods	17.70	17.77	16.72	16.49	17.70	17.77
Financial Intermediation	7.23	7.66	6.76	7.10	7.23	7.66
Real Estate Renting and Business Activity	11.55	11.89	10.76	10.91	11.55	11.89
Public Administration, Defense, and Social Security	4.33	4.25	4.35	4.22	4.33	4.25
Other Services	9.64	9.75	10.51	10.51	9.64	9.75

Source: Author's calculations from Table 1 and Table 2.

As noted earlier, TRAD yields perverse results by tending to overestimate (underestimate) the growth contribution of an industry with a low (high) relative price. This can be seen in Table 4 by comparing TRAD and the PGE component of GEN. Recall from (22) that the TRAD growth contribution of an industry is $\left(s_{t-1}^{j}/r_{t-1}^{j}\right)g_{t}^{j}$ while for the same industry the PGE component of the GEN growth contribution is $s_{t-1}^{j}g_{t}^{j}$. Hence, TRAD raises the growth contribution of an industry above PGE ($t = 2013$, Table 4) when relative price is low or $r_{t-1}^{j} < 1$ ($t - 1 = 2012$, Table 2). Conversely, TRAD lowers the growth contribution of an industry below PGE ($t = 2013$, Table 4) when relative price is high or $r_{t-1}^{j} > 1$ ($t - 1 = 2012$, Table 2). Unfortunately, these perverse results are embedded in current practice given that the TRAD growth contribution formula is so far the only "known" formula when GDP is in constant prices and, thus, is presently the "official" formula.[12]

Table 4 shows that TRAD and GEN decompositions yield sums of industry growth contributions equal to the economy's GDP growth of 7.18 percent in 2013. However, TRAD ignores *changes* in industry relative prices so that the growth contribution of an industry is given only by the single row component in the first column. In contrast, by taking into account changes in relative prices, the GEN growth contribution of an industry is the row sum of PGE, GPIE, and RPE. Consequently, when these contributions are added to get the major sector contributions, TRAD and GEN will yield different results as summarized below.

The percentage point growth contributions in Table 4 reveal that for the *Agriculture, Hunting, Forestry, and Fishing Sector*, TRAD yields 0.121 while GEN yields 0.201. These results show that TRAD understates this sector's growth contribution by underestimating GEN's PGE and excluding the positive GPIE and RPE components. A major factor for this understatement is that in this sector, *agriculture and forestry* contributed 0.106 percentage points according to TRAD which is much lower than the 0.184 percentage points contribution according to GEN due to a PGE larger than 0.106 and positive PGIE and RPE (i.e., rising relative prices) in agriculture.

For the *Industry Sector*, TRAD yields 2.978 while GEN yields 2.127. In this case, TRAD overstates this sector's growth contribution by overestimating GEN's PGE and excluding the

[12] For example, the official *Philippine Development Plan 2011-2016* used the TRAD formula in Figures 2.1, 2.2, 2.3, and 2.4, pp. 36-41.

negative GPIE and RPE components. A significant source for this overstatement is that in this sector, *manufacturing* contributed 2.269 percentage points according to TRAD which is much higher than the 1.317 percentage points contribution according to GEN due to a PGE smaller than 2.269 and negative PGIE and RPE (i.e., falling relative prices) in manufacturing. Moreover, it is interesting to note a *sign* reversal in the contribution of *mining and quarrying* from *positive* according to TRAD to *negative* according to GEN because of negative PGIE and RPE in mining and quarrying. Furthermore, there a *size* reversal in the contribution of *construction* from a *smaller* positive according to TRAD to a *larger* positive according to GEN because of positive PGIE and RPE in construction.

Table 4. TRAD and GEN Industry Contributions to Philippine GDP Growth

	TRAD	**GEN**			
	GDP growth	**PGE**	**GPIE**	**RPE**	**GDP growth**
	(percent)	(percent)	(percent)	(percent)	(percent)
	2013	2013	2013	2013	2013
		(1)	(2)	(3)	(1)+(2)+(3)
Philippine GDP percent growth	**7.18**	**7.11**	**-0.01**	**0.08**	**7.18**
Industry percentage point growth contribution					
AGRICULTURE, HUNTING, FORESTRY, AND FISHING SECTOR	**0.121**	**0.130**	**0.001**	**0.071**	**0.201**
Agriculture and forestry	0.106	0.117	0.001	0.066	0.184
Fishing	0.015	0.014	0.000	0.004	0.017
INDUSTRY SECTOR	**2.978**	**2.890**	**-0.065**	**-0.698**	**2.127**
Mining and Quarrying	0.013	0.014	-0.001	-0.090	-0.078
Manufacturing	2.269	2.108	-0.074	-0.717	1.317
Construction	0.529	0.574	0.012	0.130	0.717
Electricity Gas and Water Supply	0.167	0.174	0.000	-0.003	0.170
SERVICE SECTOR	**4.082**	**4.096**	**0.051**	**0.706**	**4.853**
Transport Communication and Storage	0.428	0.363	-0.004	-0.068	0.291
Trade and Repair of Vehicles, Personal, and Household Goods	0.948	1.003	0.018	0.326	1.347
Financial Intermediation	0.854	0.913	0.009	0.067	0.988
Real Estate Renting and Business Activity	0.935	1.005	0.015	0.168	1.187
Public Administration, Defense, and Social Security	0.166	0.166	0.002	0.061	0.229
Other Services	0.751	0.689	0.008	0.113	0.810

Source: Author's calculations from GDP in Table 1 of GEN growth components given by *pure growth effect* (PGE), *growth-price interaction effect* (GPIE), and *relative price effect* (RPE) in (15) to (17) and of TRAD growth contributions in (20).

Finally, for the *Service Sector*, TRAD yields 4.082 while GEN yields 4.853. It appears that TRAD understates this sector's growth contribution by underestimating GEN's PGE and excluding the positive GPIE and RPE components. The major factor for this understatement is that TRAD underestimated the GEN contributions of all industries by missing their positive PGIE and RPE components, except for *transport, communication and storage*. In the case of the latter industry, TRAD showed a growth contribution of 0.428 percentage points while GEN showed only 0.291 because of negative GPIE and RPE.

V. Summary and Conclusion

Considering that GDP deflators differ between industries and also differ from the economy-wide GDP deflator, this paper argues that real GDP of industries are not necessarily measured in *homogeneous* units and, therefore, questions TRAD GDP level aggregation by simple addition, i.e., without weights, of industry real GDP. As an alternative, this paper's GEN GDP level aggregation applies relative prices as weights of real GDP of industries to convert them into PPP values that are in homogeneous units–with the economy's GDP as numeraire–for additivity. For comparison, GEN and TRAD are applied to Philippine GDP in constant prices to show that the TRAD framework is deficient and misleading for overlooking the significant role of relative prices recognized by the GEN framework.

Given that relative prices change year to year and differ between industries, TRAD and GEN growth contributions of the same industry will be unequal. By ignoring *changes* in relative prices, TRAD is deficient for its inability to determine the "reallocation effects" on GDP growth of relative price changes that are measured by GEN industry growth contributions. A further deficiency is that TRAD takes into account the *level* of relative price in an industry's growth contribution in a *perverse* way by raising the growth contribution of an industry with a low relative price or, conversely, lowering the growth contribution of an industry with a high relative price. These results are contrary to the basic production principle that resources tend to be reallocated to industries with rising relative prices and, thus, promote their growth and raise their contributions to the economy's overall growth.

In the above light, the TRAD framework is illogical in theory and, therefore, misleading in practice. However, this paper's GEN framework provides a theoretically consistent and practicable alternative to TRAD for level aggregation and growth decomposition of GDP in countries, like the Philippines, where GDP is measured in constant prices.

Appendix
Additivity of GDP in Constant Prices

Let there be $j = 1, 2, \cdots, M$ industries and let each j produce $k = 1, 2, \cdots, N$ commodities over a period $t = 1, 2, \cdots, T$. Each commodity has a price p_t^{kj} and quantity q_t^{kj}. Moreover, denote the *fixed* base period by b. Hence, GDP in current prices in b and in any t are,

$$(1) \quad Y_b^j \equiv \sum\nolimits_k q_b^{kj} p_b^{kj} \quad ; \quad Y_b = \sum\nolimits_j Y_b^j \quad ; \quad Y_t^j \equiv \sum\nolimits_k q_t^{kj} p_t^{kj} \quad ; \quad Y_t = \sum\nolimits_j Y_t^j \,.$$

Let the GDP Paasche price indexes linking b to t be $P_{b,t}^P$ for the overall economy and $P_{b,t}^{jP}$ for an industry. Using the prices and quantities in (1), these indexes are, by definition,

$$(2) \quad P_{b,t}^P \equiv \frac{\sum_j \sum_k q_t^{kj} p_t^{kj}}{\sum_j \sum_k q_t^{kj} p_b^{kj}} \quad ; \quad P_{b,t}^{jP} \equiv \frac{\sum_k q_t^{kj} p_t^{kj}}{\sum_k q_t^{kj} p_b^{kj}} \;;$$

$$(3) \quad P_{b,t}^P = \sum\nolimits_j s_b^{jP} P_{b,t}^{jP} \quad ; \quad s_b^{jP} \equiv \frac{\sum_k q_t^{kj} p_b^{kj}}{\sum_j \sum_k q_t^{kj} p_b^{kj}} \quad ; \quad \sum\nolimits_j s_b^{jP} = \frac{\sum_j \sum_k q_t^{kj} p_b^{kj}}{\sum_j \sum_k q_t^{kj} p_b^{kj}} = 1 \,.$$

Let X_t and X_t^j represent real GDP of the economy and an industry in t measured in period b prices. Since b is a *fixed* base, $P_{b,t}^P$ and $P_{b,t}^{jP}$ are *direct* Paasche price indexes. In this case, X_t and X_t^j are the economy's and an industry's GDP in *constant* prices. These are computed by,

$$(4) \quad X_t \equiv \frac{Y_t}{P_{b,t}^P} = \frac{\sum_j \sum_k q_t^{kj} p_t^{kj}}{\left(\frac{\sum_j \sum_k q_t^{kj} p_t^{kj}}{\sum_j \sum_k q_t^{kj} p_b^{kj}} \right)} = \sum\nolimits_j \sum\nolimits_k q_t^{kj} p_b^{kj} \;;$$

$$(5) \quad X_t^j \equiv \frac{Y_t^j}{P_{b,t}^{jP}} = \frac{\sum_k q_t^{kj} p_t^{kj}}{\left(\frac{\sum_k q_t^{kj} p_t^{kj}}{\sum_k q_t^{kj} p_b^{kj}} \right)} = \sum\nolimits_k q_t^{kj} p_b^{kj} \;;$$

$$(6) \quad X_t \equiv \frac{Y_t}{P_{b,t}^P} = \sum\nolimits_j X_t^j = \sum\nolimits_j \frac{Y_t^j}{P_{b,t}^{jP}} = \sum\nolimits_j \sum\nolimits_k q_t^{kj} p_b^{kj} \,.$$

The result in (6) is the "additivity" of real GDP in constant prices. That is, the economy's real GDP equals the *simple* sum of the real GDP of all sectors or industries. However, as argued in the text, (6) is objectionable because $\sum_j \frac{Y_t^j}{P_{b,t}^{jP}}$ involves summation of non-homogeneous units because, while Y_t^j is homogeneous, $P_{b,t}^{jP}$ is different between industries. Thus, this paper proposes as an alternative the *weighted* sum given by,

$$(7) \quad X_t \equiv \frac{Y_t}{P_{b,t}^P} = \sum\nolimits_j r_t^j X_t^j = \sum\nolimits_j \left(\frac{P_{b,t}^{jP}}{P_{b,t}^P} \right) \frac{Y_t^j}{P_{b,t}^{jP}} = \sum\nolimits_j \frac{Y_t^j}{P_{b,t}^P} \quad ; \quad r_t^j \equiv \frac{P_{b,t}^{jP}}{P_{b,t}^P} \,.$$

Clearly, (7) is a sum of homogeneous "PPP" values of industries as explained in the text.

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