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Jesus C. Dumagan*

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Abstract

The traditional (TRAD) level aggregation and growth decomposition of GDP in constant prices—presently implemented by the Philippine Statistics Authority (PSA)—do not have real prices. However, a generalized exactly additive decomposition (GEAD) framework illustrated in this paper uses real prices—ratios of sectoral GDP deflators to the overall GDP deflator—to convert the GDP of sectors to the same units for GDP level aggregation prior to GDP growth decomposition. In GEAD, a sector's contribution equals a pure growth effect (PGE) from the sector's GDP growth—which equals PGE in TRAD—plus a price change effect (PCE) from a change in the sector's real price, which could be positive, zero, or negative. However, the sum of PCE for all sectors is zero, so GDP growth equals the sum of PGE in TRAD and GEAD. But the zero-sum of PCE does not justify ignoring PCE because this paper shows each sector's PCE is statistically significantly different from zero over time—negative for Agriculture and Services but positive for Industry—in Philippine GDP from 2001 Q1 to 2019 Q3. Thus, with no PCE, TRAD yields only PGE, which is misleading because it is just one part of a sector's growth contribution. Therefore, PSA should consider replacing TRAD with GEAD in practice.

Keywords: Real prices, GDP level aggregation, GDP growth decomposition, index numbers

JEL classification: C43, O47

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GDP growth decomposition starts from GDP level aggregation, but in the case of GDP in constant prices, the traditional (TRAD) level aggregation is the more restrictive of two ways and, thus, is a questionable basis for growth decomposition. TRAD aggregation is based on the additivity of GDP in constant prices dictated by the underlying GDP deflator, which is a fixed-base Paasche price index. However, the ratios of sectoral GDP deflators to the economy's GDP deflator may be used as *real prices* to convert the GDP of all sectors to the same units for additivity (Tang & Wang, 2004). Moreover, Dumagan (2013) noted that Tang and Wang's GDP level aggregation and growth decomposition do not depend on the deflator formula and, thus, called their framework a generalized exactly additive decomposition (GEAD).[1]

In the above light, this paper presents the two ways of aggregating the level of GDP in constant prices, which are TRAD without real prices and GEAD with real prices. The corresponding TRAD and GEAD decompositions of GDP growth show that in TRAD, a sector's contribution to GDP growth consists only of a pure growth effect (PGE) from the growth of the sector's GDP (Denison, 1962). In contrast, a sector's contribution in GEAD consists of PGE plus a price change effect (PCE) from the change in the sector's real price.

Given GDP in constant prices, the PGE of a sector is the same in TRAD and GEAD. The difference is in PCE, which is uniformly zero for all sectors in TRAD because real prices are absent. However, PCE could be positive, zero, or negative in GEAD, where the sum of PCE across all sectors equals zero. The end result is that TRAD and GEAD yield the same growth in the economy's GDP, equal to the sum of PGE across all sectors.

However, the fact that the sum of PCE is zero does not justify ignoring PCE because each sector's PCE in GEAD is statistically significantly different from zero over time—negative for Agriculture and Services but positive for Industry—as shown by the Philippine GDP from 2001 Q1 to 2019 Q3. Thus, without PCE, TRAD—presently implemented by the Philippine Statistics Authority (PSA)—yields only PGE, which is misleading because it is just one part of a sector's growth contribution. Therefore, PSA should consider implementing GEAD in place of TRAD.

[1] The simple sum of GDP in chained prices of sectors does not equal the aggregate, which is the non-additivity property of this GDP (Balk, 2010). In this case, Tang and Wang (2004), originated real prices as weights of GDP in chained prices of sectors so that their weighted sum equals the aggregate GDP, for example, in Canada and the United States. However, Dumagan (2013) pointed out that Tang and Wang's GDP aggregation applies generally to any real GDP and, therefore, to Philippine GDP in constant prices. For the above reason, Dumagan coined the acronym GEAD to describe Tang and Wang's decomposition of aggregate labor productivity—that is, GDP over total labor employment—growth that yields GDP growth by factoring out labor.

Comparing TRAD and GEAD Aggregations of GDP Level[2]

First, define nominal GDP by letting sectors $n = 1,2,\cdots,N$ produce commodities $k = 1,2,\cdots,K$ at time t with prices, p_{kn}^{t}, and quantities, q_{kn}^{t}. Thus,

$$\text{GDP in current prices of a sector} \equiv Y_n^t = \sum_{k=1}^{K} p_{kn}^{t}\, q_{kn}^{t}\,. \tag{1}$$

GDP in current prices is additive. Therefore,

$$\text{GDP in current prices of the economy} \equiv Y^t = \sum_{n=1}^{N} Y_n^t = \sum_{n=1}^{N}\sum_{k=1}^{K} p_{kn}^{t}\, q_{kn}^{t}\,. \tag{2}$$

To define the Paasche price indexes (Fisher, 1922) used as GDP deflators, let prices be p_{kn}^{b} in the year b. Treat p_{kn}^{b} as a constant by making b a fixed base year. Hence, by definition,

$$\text{Paasche price index of a sector} \equiv P_n^{bt} = \frac{\sum_{k=1}^{K} p_{kn}^{t}\, q_{kn}^{t}}{\sum_{k=1}^{K} p_{kn}^{b}\, q_{kn}^{t}}\ ; \tag{3}$$

$$\text{Paasche price index of the economy} \equiv P^{bt} = \frac{\sum_{n=1}^{N}\sum_{k}^{K} p_{kn}^{t} q_{kn}^{t}}{\sum_{n=1}^{N}\sum_{k}^{K} p_{kn}^{b} q_{kn}^{t}}\,. \tag{4}$$

It may be noted that the aggregate index, P^{bt}, has the consistency-in-aggregation (CIA) property (Diewert, 1978) because it can be expressed as the weighted sum of its subaggregate indexes, P_n^{bt}, where the weights sum to 1. CIA follows from (3) and (4) because

$$P^{bt} = \sum_{n=1}^{N}\left(\frac{\sum_{k}^{K} p_{kn}^{b} q_{kn}^{t}}{\sum_{n=1}^{N}\sum_{k}^{K} p_{kn}^{b} q_{kn}^{t}}\right) P_n^{bt} \quad ; \quad \sum_{n=1}^{N}\left(\frac{\sum_{k}^{K} p_{kn}^{b} q_{kn}^{t}}{\sum_{n=1}^{N}\sum_{k}^{K} p_{kn}^{b} q_{kn}^{t}}\right) = 1\,. \tag{5}$$

The above CIA property underlies the "additivity" property of GDP in constant prices where the economy's GDP equals the simple sum of the GDP of all sectors, as shown in (6) and (7).

GDP in constant prices is obtained by deflating GDP in current prices in (1) and (2) by their corresponding fixed-base Paasche price indexes in (3) and (4). Deflation yields,[3]

$$\text{GDP in constant prices of a sector} \equiv X_n^t \equiv \frac{Y_n^t}{P_n^{bt}} = \sum_{k=1}^{K} p_{kn}^{b} q_{kn}^{t}\ ; \tag{6}$$

[2] The following analysis applies TRAD and GEAD level aggregation and growth decomposition only to the supply or production side of GDP. However, the demand or expenditure side is examined in the Appendix of this paper where it is shown that the application of TRAD and GEAD are ill-advised based on PSA expenditure data.

[3] Value change index decomposition (Fisher, 1922) yields two ways of computing GDP in constant prices. One way is to multiply base-year GDP by a fixed-base Laspeyres quantity index. The other way, shown by (6) and (7), is to deflate (i.e., divide) nominal GDP by a fixed-base Paasche price index.

$$\text{GDP in constant prices of the economy} \equiv X^t \equiv \frac{Y^t}{P^{bt}} = \sum_{n=1}^{N}\sum_{k=1}^{K} p_{kn}^b q_{kn}^t \,. \tag{7}$$

It may be seen in (6) and (7) that $Y_n^t = P_n^{bt} X_n^t$ and $Y^t = P^{bt} X^t$. These results and the additivity of nominal GDP in (2), that is, $Y^t = \sum_{n=1}^{N} Y_n^t$, imply that there are two ways of aggregating GDP in constant prices given by

$$X^t = \sum_{n=1}^{N} \frac{P_n^{bt}}{P^{bt}} X_n^t = \sum_{n=1}^{N}\sum_{k=1}^{K} p_{kn}^b q_{kn}^t = \sum_{n=1}^{N} X_n^t \,. \tag{8}$$

Applying this paper's terminology to (8), $X^t = \sum_{n=1}^{N} X_n^t$ is the TRAD aggregation while $X^t = \sum_{n=1}^{N} \frac{P_n^{bt}}{P^{bt}} X_n^t$ is the GEAD aggregation. It is clear from (8) that TRAD and GEAD yield exactly the same aggregate GDP at constant prices. However, TRAD is restrictive by ignoring P_n^{bt}/P^{bt} while GEAD is general for the opposite reason by recognizing P_n^{bt}/P^{bt}.

Comparing TRAD and GEAD Decompositions of GDP Growth

Let t change from 0 to 1. In this case, TRAD yields the relative change in the economy's GDP given by

$$\frac{X^1}{X^0} = \frac{\sum_{n=1}^{N} X_n^1}{\sum_{n=1}^{N} X_n^0} = \sum_{n=1}^{N} w_{X_n}^0 \frac{X_n^1}{X_n^0} \,; \tag{9}$$

$$w_{X_n}^0 \equiv \text{share of GDP in constant prices} = \frac{X_n^0}{X^0} \quad ; \quad \sum_{n=1}^{N} w_{X_n}^0 = 1 \,. \tag{10}$$

It follows from (9) and (10) that the TRAD decomposition of GDP growth in constant prices is

$$\frac{X^1}{X^0} - 1 = \sum_{n=1}^{N} w_{X_n}^0 \left(\frac{X_n^1}{X_n^0} - 1\right) \quad ; \quad \text{Pure Growth Effect (PGE)} \equiv w_{X_n}^0 \left(\frac{X_n^1}{X_n^0} - 1\right). \tag{11}$$

From (11), a sector's contribution to GDP growth in TRAD comes only from the growth of the sector's GDP defined by PGE. TRAD is presently implemented by statistical agencies—like the Philippine Statistics Authority—in countries with GDP at constant prices.

To simplify notation in deriving the GEAD decomposition of GDP growth, denote real price as r_n^{bt} so that from (8),

$$\text{real price} \equiv r_n^{bt} = \frac{P_n^{bt}}{P^{bt}} \,. \tag{12}$$

Note that r_n^{bt} could be greater than, equal to, or less than 1. Following Diewert's procedure (2015), (8) and (12) yield the GEAD relative change in the economy's GDP given by

$$\frac{X^1}{X^0} = \frac{\sum_{n=1}^{N} r_n^{b1} X_n^1}{\sum_{n=1}^{N} r_n^{b0} X_n^0} = \sum_{n=1}^{N} \left[\frac{r_n^{b0} X_n^0}{\sum_{n=1}^{N} r_n^{b0} X_n^0}\right] \frac{r_n^{b1}}{r_n^{b0}} \frac{X_n^1}{X_n^0} = \sum_{n=1}^{N} s_{Y_n}^0 \frac{r_n^{b1}}{r_n^{b0}} \frac{X_n^1}{X_n^0} = \sum_{n=1}^{N} w_{X_n}^0 \, r_n^{b1} \frac{X_n^1}{X_n^0} . \tag{13}$$

Using (6), (7), and (12), the term inside square brackets in (13) becomes $s_{Y_n}^0$, the share of GDP in current prices, as shown by

$$\frac{r_n^{b0} X_n^0}{\sum_{n=1}^{N} r_n^{b0} X_n^0} = \frac{P_n^{b0} X_n^0}{\sum_{n=1}^{N} P_n^{b0} X_n^0} = \frac{Y_n^0}{Y^0} \equiv s_{Y_n}^0 \quad ; \quad \sum_{n=1}^{N} s_{Y_n}^0 = 1 . \tag{14}$$

The summation with $s_{Y_n}^0$ in (13) was first applied by Tang and Wang (2004) to GDP in chained prices in Canada and the United States, but Dumagan (2013) noted that it generally applies to any GDP in chained or in constant prices. However, if GDP is in constant prices, Dumagan (2018, 2019) rewrote (13) using $w_{X_n}^0$, the share of GDP in constant prices, by invoking the additivity of this GDP in (8), which yields

$$\frac{s_{Y_n}^0}{r_n^{b0}} = \frac{Y_n^0 / P_n^{b0}}{Y^0 / P^{b0}} = \frac{X_n^0}{X^0} \equiv w_{X_n}^0 \quad ; \quad \sum_{n=1}^{N} w_{X_n}^0 = 1 . \tag{15}$$

The result is the summation with $w_{X_n}^0$ in (13), which yields this paper's GEAD decomposition of growth of GDP in constant prices given by

$$\frac{X^1}{X^0} - 1 = \sum_{n=1}^{N} w_{X_n}^0 \left(r_n^{b1} \frac{X_n^1}{X_n^0} - 1\right) = \sum_{n=1}^{N} w_{X_n}^0 \left(\frac{X_n^1}{X_n^0} - 1\right) + \sum_{n=1}^{N} w_{X_n}^0 \left(r_n^{b1} - 1\right) \frac{X_n^1}{X_n^0} . \tag{16}$$

The GEAD decomposition in (16) has the advantage over the TRAD decomposition in (11) by recognizing the roles of both additivity and real prices in the growth of GDP in constant prices.

It appears in (16) that a sector's contribution to the economy's GDP growth equals PGE, which is the same in TRAD, plus PCE. These are defined by

$$\text{Pure Growth Effect (PGE)} \equiv w_{X_n}^0 \left(\frac{X_n^1}{X_n^0} - 1\right) ; \tag{17}$$

$$\text{Price Change Effect (PCE)} \equiv w_{X_n}^0 \left(r_n^{b1} - 1\right) \frac{X_n^1}{X_n^0} . \tag{18}$$

Therefore, the difference between TRAD and GEAD is the missing PCE in TRAD. However, aggregate growth of GDP in constant prices is only from the growth of real GDP of sectors because (16) to (18) yield:

$$\text{Sum of PCE} \equiv \sum_{n=1}^{N} w_{X_n}^{0} \, (r_n^{b1} - 1) \frac{X_n^1}{X_n^0} = \frac{X^1}{X^0} - \sum_{n=1}^{N} w_{X_n}^{0} \frac{X_n^1}{X_n^0} = \frac{X^1}{X^0} - \frac{X^1}{X^0} = 0 \, ; \tag{19}$$

$$\frac{X^1}{X^0} - 1 = \sum_{n=1}^{N} w_{X_n}^{0} \left(\frac{X_n^1}{X_n^0} - 1 \right) \equiv \text{Sum of PGE.} \tag{20}$$

But (19) does not justify ignoring PCE because doing so understates (overstates) the contributions of sectors with rising (falling) real prices when r_n^{b1} is greater (less) than 1 during any two adjoining periods (i.e., 0 and 1) so that PCE is positive (negative) in (18).

Analytically, making PCE uniformly zero for all sectors in TRAD is equivalent to assuming that in (8), (12), and (16), the real price is unitary. That is,

$$r_n^{bt} = \frac{P_n^{bt}}{P^{bt}} = 1, \text{ all } n \text{ and } t \, . \tag{21}$$

However, the assumption by TRAD in (21) is true in the base period when $t = b$ because (3) and (4) yield $P_n^{bb} = P^{bb} = 1$ but not necessarily when $t \neq b$.

Supposing that (21) is true when $t \neq b$, implies a falsity because

$$P_n^{bt} = \frac{Y_n^t}{X_n^t} \text{ and } P^{bt} = \frac{Y^t}{X^t} \text{ by definition. So } P_n^{bt} = P^{bt} \implies \frac{Y_n^t}{X_n^t} = \frac{Y^t}{X^t} \implies \frac{Y_n^t}{Y^t} = \frac{X_n^t}{X^t} \, . \tag{22}$$

The falsity is that the last equality in (22) is true when $t = b$—because in the base period, nominal and real values are the same—but not necessarily true when $t \neq b$. This falsity makes TRAD questionable. By implication, GEAD is acceptable by letting $r_n^{bt} \neq 1$ or for PCE to be positive, zero, or negative.

Visualizing the Difference PCE Makes

The following presentation compares PGE and PCE between TRAD and GEAD from Philippine GDP in current prices and constant 2018 prices from 2001 Q1 to 2019 Q3.

Table 1 presents a cross-section decomposition of Philippine GDP growth during the year 2018 Q3 to 2019 Q3 into the PGE and PCE contributions of detailed industries. The year-on-year (i.e., same quarter from year *t* to year *t*+1) calculations in Table 1 were implemented from 2001 Q1 to 2019 Q3, and the year-on-year PGE and PCE contributions of industries were aggregated to obtain Agriculture, Industry, and Services sector values. Finally, the means of the above time series values of sectoral PGE and PCE, comprising 75 observations for each sector, were tested for their statistical significance, as reported in Table 2.

Table 1

GDP Growth in the Philippines, 2018 Q3 - 2019 Q3

		PGE		PCE		GDP growth = PGE + PCE	
Growth contributions by industry (percentage points)		TRAD	GEAD	TRAD	GEAD	TRAD	GEAD
Agriculture	Agriculture, forestry, and fishing	0.2622	0.2622	0.0000	-0.2934	0.2622	-0.0312
Industry	Mining and quarrying	-0.0211	-0.0211	0.0000	0.1764	-0.0211	0.1553
	Manufacturing	0.2406	0.2406	0.0000	-0.1079	0.2406	0.1328
	Electricity, steam, water and waste management	0.2585	0.2585	0.0000	-0.8245	0.2585	-0.5660
	Construction	1.2722	1.2722	0.0000	0.2336	1.2722	1.5057
Services	Wholesale and retail trade; repair of motors	1.6320	1.6320	0.0000	-0.0402	1.6320	1.5917
	Transportation and storage	0.2809	0.2809	0.0000	0.2623	0.2809	0.5432
	Accommodation and food service activities	0.1377	0.1377	0.0000	0.0264	0.1377	0.1641
	Information and communication	0.1430	0.1430	0.0000	0.1043	0.1430	0.2473
	Financial and insurance activities	1.0631	1.0631	0.0000	-0.0952	1.0631	0.9679
	Real estate and ownership of dwellings	0.3897	0.3897	0.0000	-0.0092	0.3897	0.3805
	Professional and business services	0.0598	0.0598	0.0000	0.3345	0.0598	0.3943
	Public admin and defense; compulsory activities	0.3724	0.3724	0.0000	-0.0024	0.3724	0.3700
	Education	0.0312	0.0312	0.0000	0.1263	0.0312	0.1575
	Human health and social work activities	0.1088	0.1088	0.0000	0.0686	0.1088	0.1774
	Other services	0.0979	0.0979	0.0000	0.0405	0.0979	0.1384
	GDP growth	**6.3289**	**6.3289**	**0.0000**	**0.0000**	**6.3289**	**6.3289**

Source: Author's calculations from GDP data from the Philippine Statistics Authority.

Table 2

Tests of Significance of Means

			Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]		t
A	TRAD PGE = GEAD PGE	Agriculture	0.39	0.05	0.40	0.30	0.48	8.43
		Industry	1.58	0.11	0.98	1.35	1.80	13.97
		Services	3.54	0.13	1.09	3.29	3.79	28.26
B	GEAD PCE	Agriculture	-0.51	0.12	1.01	-0.75	-0.28	-4.41
		Industry	1.99	0.17	1.47	1.65	2.32	11.67
		Services	-1.47	0.13	1.17	-1.74	-1.20	-10.92
C	GEAD PGE + GEAD PCE	Agriculture	-0.12	0.11	0.93	-0.33	0.09	-1.13
		Industry	3.56	0.17	1.49	3.22	3.91	20.76
		Services	2.07	0.22	1.88	1.64	2.50	9.56
		Economy	**5.51**	**0.20**	**1.70**	**5.12**	**5.90**	**28.15**

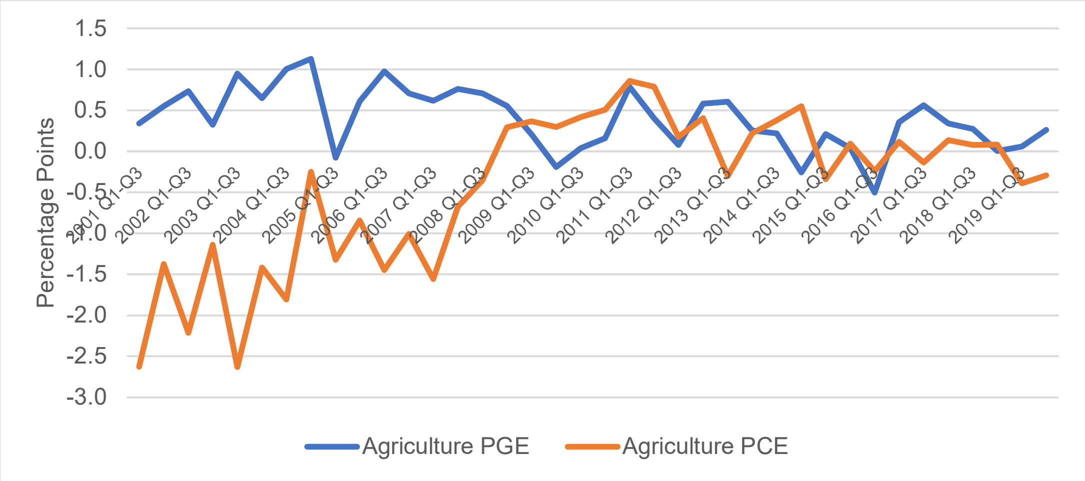
Source: Author's calculations from GDP data (2001 Q1 to 2019 Q3) from the Philippine Statistics Authority.

In Table 2, the means tests show that all the PGE by sector in row A as well as all the PCE by sector in row B are statistically significantly different from zero. Notice that because the sum of PCE is zero across sectors, the sum of the means of PCE across sectors also equals zero.

The time series values of PGE and PCE tested in Table 2 are plotted by sector in Figures 1 to 3. The PGE and PCE for the entire economy are plotted in Figure 4.

Figure 1

PGE and PCE Contributions to Philippine GDP Growth by Agriculture

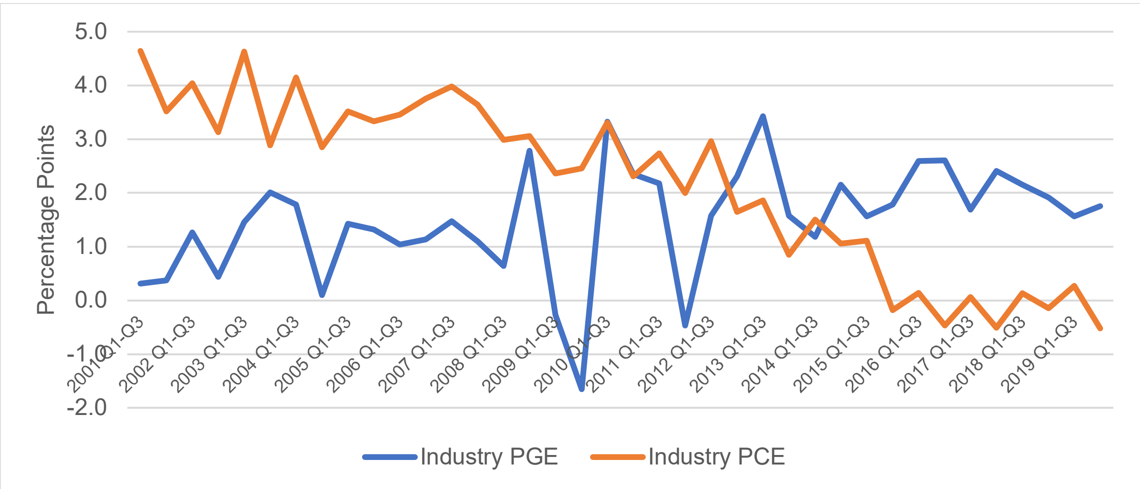


Source: Author's calculations from GDP data (2001 Q1 to 2019 Q3) from the Philippine Statistics Authority.

Agriculture PGE was mostly positive, whereas Agriculture PCE was mostly negative during 2001 Q1–2019 Q3. Thus, in Table 2, the mean of Agriculture PGE (0.39) and mean of Agriculture PCE (-0.51) are statistically significant, with t-values of 8.43 and -4.41.

Figure 2

PGE and PCE Contributions to Philippine GDP Growth by Industry

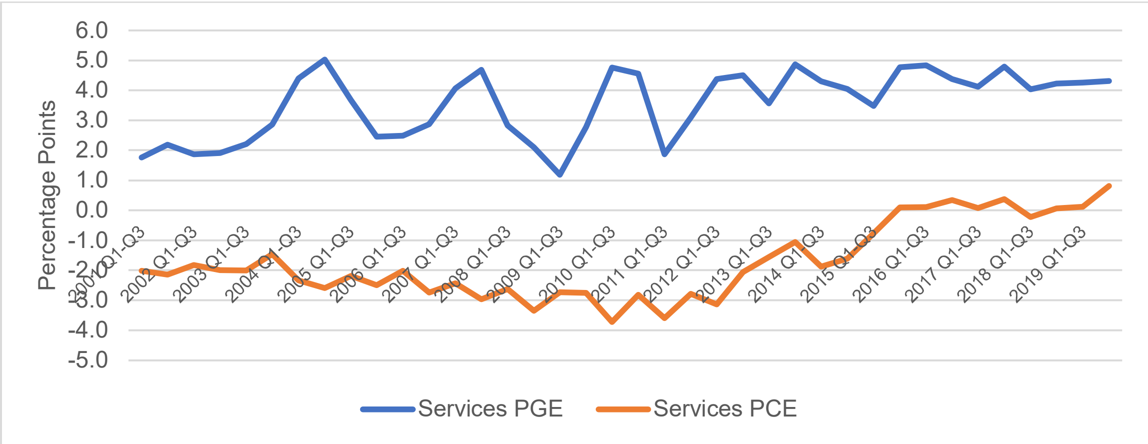


Source: Author's calculations from GDP data (2001 Q1 to 2019 Q3) from the Philippine Statistics Authority.

Table 2 shows that the means of Industry PGE (1.58) and Industry PCE (1.99) are statistically significantly positive, with corresponding t-values of 13.97 and 11.67.

Figure 3

PGE and PCE Contributions to Philippine GDP Growth by Services



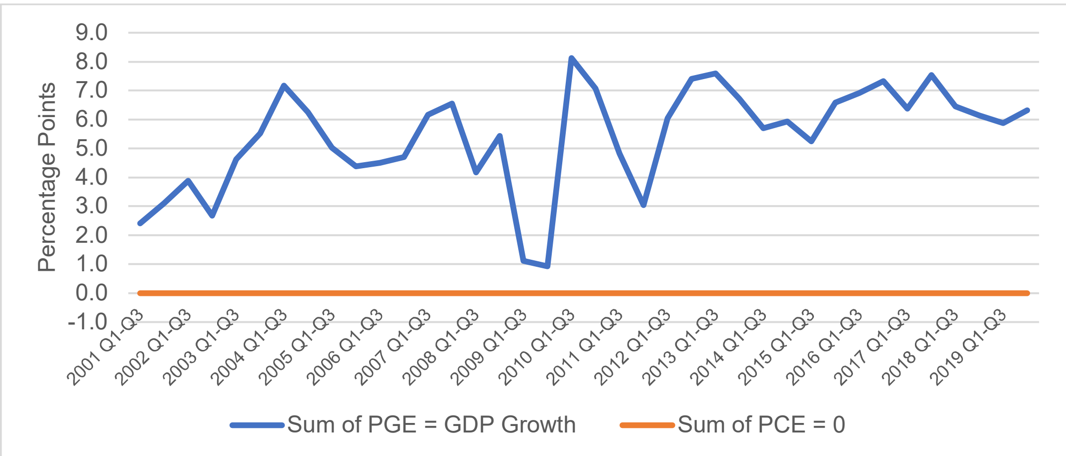
Source: Author's calculations from GDP data (2001 Q1 to 2019 Q3) from the Philippine Statistics Authority.

Services PGE stayed positive during the entire period, but Services PCE stayed negative until the last three years. This is shown in Table 2 by a positive mean of Services PGE (8.54) and a negative mean of Services PCE (-1.47) that are statistically significant with corresponding t-values of 28.26 and -10.92.

If PGE and PCE in Figures 1 to 3 are summed vertically (i.e., across sectors in each quarter), the result is the economy's PGE and PCE in Figure 4. This aggregate PGE is the same in TRAD and equals the Philippine economy's GDP growth because the sum of PCE = 0.

Figure 4

Sum of PGE and Sum of PCE Contributions to Philippine GDP Growth



Source: Author's calculations from GDP data (2001 Q1 to 2019 Q3) from the Philippine Statistics Authority.

In Table 2, the sum of PGE for the economy equals the sum of GEAD PGE + GEAD PCE for all sectors that has a statistically significant mean of 5.51% with a t-value of 28.15.

With no real prices, PCE is zero in TRAD. In GEAD, the sum of PCE for all sectors equals zero because the definitions in (5) to (12), where $w_{X_n}^t \equiv X_n^t / X^t$ and $\sum_{n=1}^{N} w_{X_n}^t = 1$, yield

$$P^{bt} = \sum_{n=1}^{N} w_{X_n}^t P_n^{bt} \quad ; \quad r_n^{bt} = \frac{P_n^{bt}}{P^{bt}} \quad ; \quad 1 = \sum_{n=1}^{N} w_{X_n}^t r_n^{bt} \quad ; \quad 0 = \sum_{n=1}^{N} w_{X_n}^t (r_n^{bt} - 1)\,. \quad (23)$$

It can be shown that (23) implies that the sum of PCE $= 0$ in (19). Therefore, in Figure 4, the growth of the economy's GDP equals only the sum of PGE in both TRAD and GEAD.

The results in (23) are fundamental for the GEAD framework. To recapitulate, the first equality is true by definition of the fixed-based Paasche price indexes that yield the sectoral GDP deflator, P_n^{bt}, and the economy-wide GDP deflator, P^{bt}. The second equality defines the real price, r_n^{bt}, of a sector's GDP that could differ from or equal 1. The third equality means that the average real price is 1, which is the real price of a "unit" of the economy's GDP being the numeraire. Finally, as the real GDP of sectors change, so will their real prices change. But sectoral real prices change relative to the economy-wide real price of 1. Therefore, sectoral real price changes average out to zero according to the last equality in (23).

At this juncture, it may be noted that TRAD underlies the finding that Philippine economic growth "deviated" (de Dios & Williamson, 2013) from the norm where resources move from Agriculture more to Industry before moving on to the Services sector. The deviation is that, as a result, growth in the Philippines "leapfrogged" from Agriculture over Industry to Services. Interestingly, this growth leapfrogging appears to be borne out by the means tests in Table 1, where the TRAD results in row A show the mean growth contributions (in percentage points) are Agriculture (0.39), Industry (1.58), and Services (3.54), which are all measured only by PGE. This appears to support leapfrogging from the fact that the growth contribution of Services exceeds that of Industry at this stage when the Philippines is still a "poor" country.

However, if the effects of changes in real prices are included, the growth contributions are those given in row C by Agriculture (-0.12), Industry (3.56), and Services (2.07) that are contrary to the leapfrogging story, noting that the mean growth contribution of Services is less than that of Industry. That is, the GEAD decomposition of growth of the Philippine economy depicts a normal pattern where resources move from Agriculture to boost Industry growth ahead of Services. In this case, however, the negative Agriculture mean growth contribution indicates that when both PGE and PCE are accounted for, the growth of this sector trends downward, although this is not statistically significant, as shown by the low t-value (-1.13) in Table 2.

Finally, to visualize the absence of PCE in TRAD and its presence in GEAD, imagine GDP as a cake. Slice the cake through the center many times to create slices of different sizes representing the GDP of sectors. Assume that the cake was baked in the base period ($t = b$) so that the real price of each slice is $r_n^{bb} = P_n^{bb}/P^{bb} = 1$ because base-period price indexes equal 1. This makes the cake flat on top because all slices start with the same height of "1." Next, let the GDP cake grow. In this case, TRAD keeps $r_n^{bt} = 1$ while GEAD allows r_n^{bt} to change from 1.

TRAD makes the cake grow while keeping it flat; thus, slices have the same thickness. GEAD allows slices to become thicker, the same, or thinner through PCE. PGE is the change in the "surface area of a slice," which is the same in TRAD and GEAD. GDP growth is the change in the "volume of the whole cake," which is also the same in TRAD and GEAD. Hence, PCE is a change in height that represents a change in the "volume of a slice" given PGE. Therefore, TRAD and GEAD cakes have different tops because the former is flat while the latter is jagged. If the GEAD cake is flattened to a height of "1", it becomes identical to the TRAD cake.

The above visualization shows that PCE changes the picture of how the economy's GDP grows with real implications because PCE—as a change in volume of a slice of the GDP cake—is a change in a sector's GDP in real terms and, therefore, should be measured.

Moreover, the above picture implies from Table 2 that Agriculture, Industry, and Services, as "GDP slices," are all increasing in the surface area because the means of their PGE are all positive (row A). However, the Agriculture and Services slices are getting thinner because the means of their PCE are both negative, whereas the Industry slice is getting thicker by showing a positive mean PCE (row B). Finally, in terms of changing volume, Agriculture appears to be shrinking, whereas Industry, Services, and the economy as a whole are expanding (row C).

Conclusion

This paper found that GEAD yields TRAD as a restrictive special case by having no real prices (i.e., zero PCE) and that PCE in GEAD is a statistically significant component of a sector's contribution to the growth of Philippine GDP. Thus, TRAD—presently implemented by PSA—yields only PGE, which is misleading because it is just one part of a sector's growth contribution. Therefore, PSA should consider replacing TRAD by GEAD in practice.

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Appendix

Why Computing Growth Contributions from the Demand or Expenditure Side of GDP is Ill-Advised

This paper's TRAD and GEAD frameworks for level aggregation and growth decomposition focused solely on the supply or production side of GDP. Technically, TRAD and GEAD may be applied to the demand or expenditure side. However, it is shown below that the application is ill-advised because the way the expenditure components of GDP are presently defined by PSA makes the effect of a change in an expenditure component on GDP growth analytically ambiguous.

First of all, note that there has been no mention of imports—that add to the supply of goods and services in the country—because GDP, by definition, counts only what is supplied from *within* the geographic limits (i.e., domestic) of the country (Branson, 2005; Mankiw, 2019). GDP, however, includes goods and services produced by citizens and foreigners within the country. That is, GDP measures domestic supply or production, regardless of the nationality of the producer.

Recall from (2) in the main body of this paper that GDP in current prices is denoted by Y^t, which is domestic supply as defined above. That is,

$$\text{GDP in current prices of the economy} \equiv Y^t = \text{domestic supply}\,. \tag{A-1}$$

Introducing additional notation, let

$$\text{total imports} \equiv M^t = \text{foreign supply}\,; \tag{A-2}$$

$$\text{AS} \equiv \text{aggregate supply} = \text{domestic supply} + \text{foreign supply} = Y^t + M^t\,. \tag{A-3}$$

Consider now that domestic supply faces domestic demand and foreign demand as well. Domestic demand—made possible by income generated by domestic production—comprises demand for domestically produced goods and services and for those produced abroad (i.e., imports). Foreign demand gives rise to exports of part of domestic production. So, the counterpart to (A-2) and (A-3) on the demand side may be given as

$$\text{total exports} \equiv X^t = \text{foreign demand}\,; \tag{A-4}$$

$$\text{AD} \equiv \text{aggregate demand} = \text{domestic demand} + \text{foreign demand} = Y^{dt} + X^t\,. \tag{A-5}$$

In national income accounting, AS = AD so that from (A-3) and (A-5),

$$Y^t + M^t = Y^{dt} + X^t \quad ; \quad Y^t = Y^{dt} + X^t - M^t\,. \tag{A-6}$$

In (A-6), domestic demand (Y^{dt}) may be expressed as the sum of the familiar textbook consumption demand (C^{t}), investment demand (I^{t}), and government demand (G^{t}),

$$Y^{dt} = C^{t} + I^{t} + G^{t} . \qquad \text{(A-7)}$$

The above demands are satisfied by either domestic (denoted by the superscript d) or by imported (denoted by superscript m) goods and services. That is,

$$C^{t} = C^{dt} + C^{mt} \quad ; \quad I^{t} = I^{dt} + I^{mt} \quad ; \quad G^{t} = G^{dt} + G^{mt} ; \qquad \text{(A-8)}$$

$$M^{t} = C^{mt} + I^{mt} + G^{mt} . \qquad \text{(A-9)}$$

Finally, noting the definitions in (A-3) to (A-5), combining (A-6) to (A-9) yields

$$Y^{t} = C^{dt} + I^{dt} + G^{dt} + X^{t} . \qquad \text{(A-10)}$$

This states that domestic supply (Y^{t}), which is GDP, equals the sum of domestic demands ($C^{dt} + I^{dt} + G^{dt}$) plus foreign demand (X^{t}). It is important to note that the components in the right-hand side of (A-10) are mutually exclusive that is necessary to "isolate" the effect on GDP growth (i.e., growth of Y^{t}) of a change in any one of the right-hand side components. Unfortunately, except for X^{t}, these components are not available in the "actual" GDP expenditure categories released by PSA. Thus, C^{dt}, I^{dt}, G^{dt} and the expression in (A-10) are unknown.

Therefore, in place of (A-10), PSA releases the expenditure categories that satisfy the equivalent GDP expression given by

$$Y^{t} = C^{t} + I^{t} + G^{t} + X^{t} - M^{t} . \qquad \text{(A-11)}$$

The components of (A-11) are available in current prices and in constant prices. Hence, it is technically possible to apply the TRAD and GEAD frameworks—for decomposing GDP growth into the contributions of changes in expenditure components—to (A-11) but this application is ill-advised. The reason is that (A-8) and (A-9) imply that changes in the right-hand components of (A-11) are not mutually exclusive. For example, an increase in M^{t} could be due to an increase in either one or all of C^{t}, I^{t}, and G^{t}. Moreover, even X^{t} may have imported components. In this case, the effect of a change in M^{t} on GDP growth is analytically ambiguous (i.e., cannot be isolated). This ambiguity applies in principle to the effect on GDP growth of a change in any one of the right-hand side components of (A-11).

In conclusion, TRAD and GEAD decompositions of GDP growth from the demand or expenditure side are analytically acceptable if applied to (A-10) but ill-advised if applied to (A-11). This is unfortunate because expenditure data from PSA are available only for (A-11).